ABSTRACT
The interaction between the cylinder motion and the wake is a complex feedback phenomenon in which the symmetry relationship between the wake and the cylinder motion plays a key role. Depending on the frequency of oscillation the symmetry relationship between the unforced von Karman wake and the imposed forced oscillations can induce a series of bifurcations. This detailed bifurcation behavior is the subject of study in the present work. 2D and 3D simulations are carried out for a Reynolds number Re=1000. As the inline cylinder forcing amplitude is increased, the wake undergoes a series of bifurcations and associated changes in the flow structure. Although the 2D analysis is clearly non-physical, it leads to a ‘simpler’ and more tractable model.

Detailed comparison of the 2D and 3D POD modes provides insight into the forced wake dynamics. The 3D spatial mode shapes are significantly similar to those from 2D simulations. The relative modal energy distribution captures well the wake flow complexity. The first mode contains over 90% of the flow energy in the 2D simulations. This ratio drops significantly in the 3D case to around 45%. Clearly 3D effects are very important when it comes to energy distribution between the modes. However, the predominance of the first mode seems high enough to maintain the 2D-like dynamics. The wake flow is found to undergo two main transitions with increased forcing. The first is periodic shedding to chaotic shedding. The second is chaotic shedding to half-frequency shedding caused by a period-doubling bifurcation. The 2D simulations correctly predict these bifurcations – including the type and the number of bifurcations. The results suggest that the forced wake dynamics are primarily dominated by two-dimensional rather than 3D dynamics. However, 3D effects are important in determining the exact parameter values where bifurcations occur. A previously developed low order analytical model, based on 2D simulations, is also used to predict the wake bifurcation behavior. The relevance of the low order model has interesting implications for VIV control.

INTRODUCTION
The flow around an isolated cylinder and the associated Karman wake have been the subject of study over more than a century. Following a Hopf bifurcation and onset of vortex shedding at the critical Reynolds number near Re=45 the wake behaves, dynamically, as a highly organized flow structure of low effective dimension. The phenomenon can thus be described in terms of a small number of orthogonal modes.

Computational advances have now made it possible to simulate the flow around a circular cylinder in great detail. To better understand the wake dynamics and the related vortex-induced-vibrations (VIV) problem, researchers have also studied the related problem of cylinder-wake interaction during forced cylinder oscillation (e.g. Ongoren & Rockwell (1988)). The experimental tests have shown that under forced excitation the cylinder wake undergoes a series of bifurcations resulting in wake patterns, sometimes described in terms of the number of vortices shed per cycle of oscillation, e.g. 2P, P+S, 2S etc., Williamson & Roshko (1988)). An excellent review of earlier VIV research is that by Williamson and Govardhan (2004).

Work on controlled wake dynamics and comparison of free and forced wakes has also been done by Carberry et al. (2004, 2005) among others. Besides experimental work, CFD computations of the forced wake flow have been presented, for instance, by Blackburn and Henderson (1999), Meneghini and Bearman (1995), and Dong and Karniadakis (2005). More recent work has been presented for instance by Leontini et al. (2006) and Baranyi (2009, 2008); the latter studied the flow around an orbiting cylinder as well as the case of forcing at very low Reynolds number.

While the wake itself is clearly three dimensional for high Reynolds numbers, the patterns described above are based on
essentially 2D observations. The experimental observations seem to suggest that despite the 3D nature of the wake flow, the primary bifurcations – under forced excitation – may be governed primarily by two-dimensional dynamics.

This observation led Mureithi et al. (2010, 2003, 2002) to develop a symmetry-equivariance based reduced order model of the forced wake. The model parameters were obtained from 2D CFD simulations of the cylinder wake. While the model was shown to predict the observed 2D wake flow dynamics, questions remained regarding the validity or implications of the model for the more realistic case of true 3D forced wake flow.

The present work aims to address the question of the importance of Karman wake flow three-dimensionality, for the specific case of the periodically forced wake. Cylinder excitation is limited to the inflow direction. The cylinder is considered rigid (hence undergoes rigid body oscillations). Oscillation amplitudes are limited to 35% of cylinder diameter. This is considered a reasonable limit for the validity of a reduced order model. In this study, we wish to compare the bifurcation behavior of the forced 3D wake with that of the idealized 2D wake for Re=1000. This comparison will, in turn, allow us to investigate the extent to which the 3D wake dynamics and bifurcation behavior can be predicted by the much simpler reduced order model based on a flow constrained to two dimensions.

At the outset it is evident that the simple reduced order model cannot predict the full 3D wake dynamics. The basis for comparison is the general wake dynamical behavior. In particular, the wake instabilities and bifurcations as the forcing amplitude is varied. The question is whether the wake bifurcations can be, at least qualitatively, be predicted by a simple 2D reduced order model. If successful, such a model could have interesting potential for VIV control.

**CFD COMPUTATION PARAMETERS**

**2D simulation parameters**

The two-dimensional unsteady incompressible Navier-Stokes equations were solved, in the oscillating cylinder reference frame, using a finite volume method with second order accuracy in both time and space using the commercial code Fluent. To account for the moving reference frame, time-dependent boundary conditions and a source term in the flow corresponding to the inertia term were introduced. The boundary conditions and source term are presented in eq. (1).

\[
\begin{align*}
\begin{cases}
u = U - A \omega \cos \omega t & \text{and } v = 0 & P = P_{\infty} \\
u_{\text{free}} \text{ and } v_{\text{free}} & \text{and } \frac{\partial P}{\partial y} = 0 \\
\text{Source term} = \rho A \omega^2 \sin \omega t \frac{\partial \omega}{\partial \omega} \end{cases}
\end{align*}
\]

For the lateral sides and inflow boundary the condition of a flow velocity field undisturbed by the cylinder is applied. The structured mesh consisting of 130,000 cells is shown in Fig. 1. The domain boundary was located 15D upstream, 20D laterally and 40D downstream of the cylinder.

![Structured mesh with 130 000 cells.](image)

In the computations cylinder is excited periodically in the flow direction with reduced amplitudes $A/D = 0.0-0.35$. The Reynolds number is 1000 while the forced oscillation dimensionless frequency is chosen as $St = 0.21$.

**3D simulation parameters**

The 3D unsteady simulations are carried out by using ANSYS CFX for a Reynolds number of 1000. The cylinder and the computational domain span 5 diameters. A structured mesh consisting of 6.5 million nodes (60 nodes in the spanwise direction) was used. The mesh in a typical plane of the cylinder cross-section is shown in Fig. 2. The grid has a $y^+ \leq 1$ near the cylinder surface. The large eddy simulation (LES) model is applied to simulate the turbulence in the flow. The time step is chosen as 1/500 of the vortex shedding period of the fixed cylinder (the Strouhal number, $St$, is equal to 0.21 in the current simulation, to match the 2D case). The computational domain has time-dependent inlet boundary condition and a source term as in the 2D simulations. The outlet boundary condition is set as uniform atmospheric pressure. No slip conditions are imposed at the cylinder surface with free slip condition at both transverse and span-wise boundaries.
The stationary cylinder wake
In this base reference case, the cylinder wake shows stable Karman vortex shedding in both the 2D and 3D simulations as shown in the respective force coefficient time traces in Fig. 3 and Fig. 4. As expected, the 2D simulations yield significantly higher drag and lift forces compared to the 3D case. The wake flow structures are compared in Fig. 5 and Fig. 6. Three dimensional effects lead to faster dissipation of the wake vorticity as seen in Fig. 6 and detailed further below.

**Forced excitation for A/D=0.15**
The force coefficient and lift force spectrum results for cylinder excitation with oscillation amplitude-to-diameter ratio A/D=0.15 in the flow direction are shown in Fig. 7 and Fig. 8. Clearly, in this case, there is a significant difference between the 2D and 3D cases. While the 2D case is purely periodic, the 3D wake shows strong non-periodic behavior. Note, however, that the lift frequency spectra show that in both cases the dominant dimensionless frequency is near $fD/U=0.1$. More insight may be gained by comparing the detailed wake structures in Fig. 9 and Fig. 10 which show snapshots of the wake structures.

Figures 5-10 show clearly that, as expected, the 2D simulations are not fully representative of the 3D wake flow observed in this case. For the two-dimensional case non-periodic behavior akin to that shown in Fig. 8, was also found.
Figure 7. $C_D$ and $C_L$ time evolutions in the case of 2D simulation for forced cylinder, $A/D=0.15$, $Re=1000$.

Figure 8. $C_D$, $C_L$ time evolutions and $C_L$ power spectrum, in the case of 3D simulation for forced cylinder, $A/D=0.15$, $Re=1000$.

Figure 9. Wake structure from 2D simulations for forced excitation amplitude, $A/D=0.15$, for $Re=1000$.

Figure 10. Wake structure from 3D simulations for forced excitation amplitude, $A/D=0.15$, for $Re=1000$.

Figure 11. Lift coefficient time variation from 2D simulations for $A/D=0.125$, $Re=1000$.

for a smaller forcing amplitude of $A/D=0.125$. The results, shown in Fig. 11, suggests that a key difference between the 2D and 3D results is the forcing amplitude values at which bifurcations in the wake flow structure occurs. Further discussion on the underlying bifurcations is found below.

**Forced excitation for $A/D=0.35$**

We consider next the case where the cylinder forcing amplitude is $A/D=0.35$. The 2D results are presented in Fig. 12. The lift data shows that another bifurcation has occurred in the wake flow such that the lift frequency is now predominantly at approximately half the fixed cylinder shedding frequency. Furthermore, the flow is strongly periodic for this forcing amplitude.

Figure 13 shows the corresponding results obtained from 3D simulations. Remarkably, the lift and drag time traces are very similar to the two-dimensional case. Equally interestingly, the lift and drag force magnitudes are very nearly equal for the 2D and 3D simulations. The corresponding wake structures are shown in Fig. 14 and Fig. 15, respectively. Despite the strongly three-dimensional nature of the wake in Fig. 15 it is also clear that the vorticity shedding is highly correlated. The shedding mode is $2P$ – corresponding to two pairs of vortices shed per cycle of cylinder oscillation. This shedding mode is the result of a period-doubling bifurcation (Mureithi et al., 2010) which results in wake oscillations at approximately half the fixed-cylinder shedding frequency.
Figure 12. $C_D$ and $C_L$ time evolutions in the case of 2D simulation for forced cylinder, A/D=0.35, Re=1000.

Figure 13. $C_D$ and $C_L$ time evolutions and $C_L$ spectrum, from 3D simulations for forced cylinder, A/D=0.35, Re=100.

Figure 14. Wake structure from 2D simulations for forced excitation amplitude, A/D=0.35, for Re=1000.

Figure 15. Wake structure from 3D simulations for forced excitation amplitude, A/D=0.35, for Re=1000.

External forcing and flow 3-dimensionality
Figures 16-18 provide more details of the vorticity in the cylinder wake. For the fixed cylinder, Fig. 16, the primary 2D wake structure is clearly visible in Fig. 16(b). In the axial direction, however, the wake structure is broken into cells introducing three dimensionality in the wake.

Figure 16(a). Side view of vorticity contours for the fixed cylinder (A/D=0).

Figure 16(b). Isometric view of vorticity contours for the fixed cylinder (A/D=0).

For A/D=0.15 forcing, the cylinder wake flow is drastically different, Fig. 17. The wake appears fully 3D. It is clear that for this case a 2D approximation would be quite inaccurate since flow two-dimensionality is practically absent. Fig. 17 also explains the nearly chaotic lift coefficient variation observed in Fig. 8.
The situation changes for the higher forcing amplitude of $A/D=0.35$. As seen from Fig. 18, and in comparison with Fig. 17, the 2D flow structure is once again enhanced and hence more dominant for this higher amplitude forcing. Clearly one of the reasons for this is the higher amplitude forcing which strengthens 2D flow perturbations. More importantly, however, is a period-doubling bifurcation. This bifurcation is confirmed to be essentially a 2D bifurcation as discussed below. The bifurcation further strengthens the 2D flow/vortex structures. This conclusion is also supported by the time traces and frequency spectra presented in Fig. 12 and Fig. 13.

**Wake flow POD modes**

For more detailed comparison between the 2D and 3D flows, modal decomposition of the wake flow was conducted. Proper Orthogonal Decomposition (POD) extracts a reduced number of typical spatial mode shapes with associated time evolutions. The orthogonal functions (POD modes) are optimal, in the sense that they capture the maximum energy contained in the flow field. Here, POD analysis is performed on the fluctuating flow velocity. The $x$-direction fluctuating velocity $u'$, for instance, is given by

$$u'(y,t) = u(y,t) - \langle u(y,t) \rangle$$  \hspace{1cm} (2)

where $y$ is the coordinate transverse to the flow direction and the position-dependent time average, $\langle u(y,t) \rangle$, has been removed. Eq. (3) shows the orthogonal decomposition of the $x$-velocity fluctuation $u'$. Each mode is the product of a spatial function $\Psi_k(y)$, called the *topos*, and a temporal one $a_k(t)$, called the *chronos*.

$$u'(y,t) = \sum_{k=1}^{141} a_k(t) \Psi_k(y)$$  \hspace{1cm} (3)

The POD analysis is performed using data taken from a location 10D downstream of the cylinder. Of particular interest is the level of similarity or difference between the POD modes from the 2D and 3D simulations. As mentioned earlier, the key hypothesis of the present work is that 2D modes may give a good approximation of the more complex 3D wake flow behind the forced cylinder. The POD modes provide a good basis for comparison of the two types of flows.

The first two POD modes from the 2D analysis are presented in Fig. 19. The spatial variation (*topos*) appears as a velocity profile across the cylinder wake. The temporal variation (*chronos*) is presented in the frequency domain for convenience. Fig. 20 shows the corresponding two POD modes (in projection) from the 3D simulations. The spatial mode shapes are very similar to those from 2D simulations. We note, however, that while the frequency spectrum for mode 1 is similar in the 2D and 3D cases, the second mode spectrum is clearly more affected by flow complexity in the 3D case.

Another important parameter is the relative energy distribution between the various POD modes. As shown in Fig. 21, the first mode contains over 90% of the flow energy in the 2D simulations. This ratio drops significantly in the 3D case to...
Figure 19. The first two POD modes from 2D simulations.

Figure 20. The first two POD modes from 3D simulations for the fixed cylinder (A/D=0), Re=1000.

Figure 21. Flow energy distribution in 2D case.

Figure 22. Flow energy distribution for 3D simulations.

around 45%, Fig. 22. Clearly, 3D effects are very important when it comes to energy distribution between the modes. A two-dimensional projection of the topos is shown in Fig. 20 to allow comparison with the 2D case. The actual 3D topos of mode 1 is presented in Fig. 23; recall that this is the spatial mode derived from the (streamwise) $u'$ velocity fluctuations.

Figure 23. $u'$ velocity Mode 1 topos for the fixed cylinder case; the cylinder is at extreme left.

Despite the apparent complexity of the wake flow in Fig. 16, it is clear from the POD mode of Fig. 23 that a highly coherent (vortex shedding) flow structure exists and is dominant in the flow. The corresponding (transverse) $v'$ velocity topos, Fig. 24,

Figure 24. $v'$ velocity mode 1 topos for the fixed cylinder case; the cylinder is at extreme left.
confirms this conclusion regarding the highly organized nature of the first POD mode. Three-dimensional effects are also, however, clearly apparent in the z-direction velocity fluctuations $w'$ (associated with 3D transition) as shown in Fig. 25. This explains the complex vorticity contours of Fig. 16.

Figure 25. w-velocity Mode 2 topos for the fixed cylinder case; the cylinder is at extreme left.

2D low-order model of the forced wake
In previous work (Mureithi et al., 2010, 2002), the authors presented some low-order models for the forced wake. The models are based on symmetry-equivariant bifurcation theory. Starting with the hypothesis that the wake flow is dominated by the first two POD modes (see Fig. 19), labeled K and S here, a 3rd order discrete map for the wake dynamics was developed. The development of the model may be briefly outlined as follows. Since we are interested in the dynamic modal interaction for forced cylinder motion, the mode S is assumed to be generated (or directly amplified) by the cylinder motion itself hence has the same wavelength as the K mode.

When the flow field is dominated by the K and S modes, the $u$-velocity perturbations, for instance may be written as

$$u(x, y, t) = S(t)\psi_S(y)e^{i(k_x x + \omega t)} + K(t)\psi_K(y)e^{i(k_x x + \omega t)} + \text{complex conjugate}$$  \hspace{1cm} \text{(4)}

$S(t)$ and $K(t)$ are the respective mode amplitudes while the form of the spatial component of the modes is given by the first two POD modes in Fig. 19. Introducing a discrete Poincare map representation, the evolution of the fluid state may be represented by a mapping of the form

$$u_{n+1}(x, y, t_{n+1}) = \Phi[u_n(x, y, t_n)]$$  \hspace{1cm} \text{(5)}

Limiting the flow to the two modes in eq. (4), the mapping describing the interaction between these modes in the amplitude domain is, \(\Phi(S, K) = [\phi_1(S, K), \phi_2(S, K)]^T\). Based on the symmetries of the fundamental S and K modes, Mureithi et al. (2002) arrived at a general form of the non-linear interaction equations for different wavelength ratios \(m:n\).

When the cylinder undergoes forced excitation at the vortex shedding frequency, \(m:n=1:1\), the resulting mode interaction amplitude equations are

$$\frac{dK}{dt} = (\alpha_0 + \gamma_1|S|^2 + \alpha_2|K|^2)K + \delta_0 S K^2$$

$$\frac{dS}{dt} = (1 + \beta_0 + \beta_2|S|^2 + \gamma_2 |K|^2)S + \mu_0 S K^2$$  \hspace{1cm} \text{(6)}

The model coefficients have been obtained using the 2D POD modes of Fig. 19. To investigate the effect of S mode forcing on the K mode, a simulation of the discrete form of the K-equation above, varying S as a parameter has been done. The model results may be summarized as follows (Mureithi et al., 2010). For zero forcing, the K mode solution corresponds to the natural Karman vortex shedding. For moderate S-forcing, bifurcations occur in the wake leading to complex chaotic-like behavior involving the interaction dynamics deriving from two-unstable fixed points and an outer limit circle via a homoclinic bifurcation. In the CFD results, this corresponds to the complex chaotic-like wake of Fig. 17 or the lift variation in Fig. 11. For the highest forcing, a period-doubling bifurcation is confirmed by the low-order model, in the form of 2 fixed points of the K-amplitude.

Bifurcation sequence in the forced wake
In this section we summarize the observed bifurcation behavior in tabular form. Bifurcations lead to important changes in flow structure. The comparison criteria include the forcing amplitudes at which bifurcations occur and more importantly, the type of bifurcations and resulting flow structure. The 2D predictions are compared to the expected 3D model predictions in Table 1. In the table, the predictions of the low order model are also presented.

The results may be summarized as follows. The wake undergoes two main transitions or bifurcations. The first is periodic shedding to chaotic shedding. The second is chaotic shedding to half-frequency shedding caused by period-doubling bifurcation. The 2D simulations correctly predict these bifurcations – including the type and the number of bifurcations. This confirms the validity of 2D CFD for the specific problem of the forced wake considered here. Importantly, the result confirms that the forced wake dynamics are primarily 2D, not 3D. However, 3D effects are important in determining the exact parameter values where bifurcations occur.

The CFD results are also compared with the simple model in Table 1. The comparison confirms that the very simple low order model of eq. (6) is capable, to a certain extent, of predicting the correct bifurcations and dynamics expected in the 3D wake during forced excitation.

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Table 1: Comparison of wake bifurcations predicted by 3 models

<table>
<thead>
<tr>
<th>MODEL/ FORCING</th>
<th>A/D=0 Fixed cylinder</th>
<th>A/D=0.125</th>
<th>A/D=0.15</th>
<th>A/D=0.35</th>
</tr>
</thead>
<tbody>
<tr>
<td>2D CFD</td>
<td>Classical periodic</td>
<td>Chaotic</td>
<td>P+S mode shedding; onset of period-doubling</td>
<td>Period-doubling (St=0.1)</td>
</tr>
<tr>
<td></td>
<td>Karman shedding</td>
<td>vortex shedding</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3D CFD</td>
<td>Classical</td>
<td>Chaotic</td>
<td>Period-doubling</td>
<td>Period-doubling</td>
</tr>
<tr>
<td></td>
<td>Karman shedding</td>
<td>vortex shedding</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Low order</td>
<td>Limit cycle=Classical</td>
<td>Chaotic</td>
<td>Period-doubling</td>
<td>Period-doubling</td>
</tr>
<tr>
<td>model</td>
<td>Karman shedding</td>
<td>transition via heteroclinic bifurcation</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

VIV bifurcation control
The simple low order model of eq. (6) therefore seems to work surprisingly well. Keeping in mind the complexity and three dimensionality of the cylinder wake (Figs. 16-18) it is quite surprising that such a simple low order model can predict all the important bifurcation observed in the wake. The underlying flow and forcing symmetries shed some light on why the simple model works. Note that the cylinder forcing is itself 2D.

The foregoing result also suggests a potentially effective feed-forward control approach for vortex-induced vibrations. The challenge for VIV control is the ability to control the flow field in time and space (spatio-temporal control) in order to suppress vortex shedding. We propose here a different approach. Rather than suppressing vortex shedding, forced excitation aimed at triggering a bifurcation of the Karman wake is expected to be much more effective. Much less energy should be needed since vortex shedding is not suppressed; rather the vortex shedding frequency is changed (via period-doubling to half the normal frequency). Physically, this would eliminate resonance with a structural frequency matching the vortex shedding frequency.

Conclusions
The results presented here show that, from a dynamics point of view, 2D simulations provide a good approximation of the 3D wake dynamical behavior of the forced cylinder wake. In particular, all the bifurcations in the 3D simulations are correctly predicted. Furthermore, the low order model was shown to predict the same bifurcations. This result supports the importance of symmetry considerations in deriving the low order model; alternatively, it can be argued that the system symmetry dominates the wake dynamics, which explains the predictive accuracy of the low order model.

The low order model presents a potentially effective and simple strategy for VIV control. An eigenvalue analysis confirms that reflection-symmetric S-mode forcing strongly affects the Karman shedding mode by inducing a period-doubling bifurcation. The bifurcation essentially eliminates lock-in and thus potential VIV.

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