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MINIMUM COST DESIGN OF LATERALLY LOADED WELDED RECTANGULAR CELLULAR PLATES

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1. Introduction

A cellular plate consists of two parallel face sheets welded to an orthogonal grid of ribs sandwiched between them (Fig.1.). This type of welded sandwich plates has the following advantages over plates stiffened on one side: *a./ the torsional stiffness is much larger, b./ the height of ribs can be much smaller, c./ fabricational imperfections due to the shrinkage of welds are much smaller because of the symmetry of the structure, d./ the planar surface can be more easily protected against corrosion.*

Cellular plates may be applied in ships, bridges, dock gates, light-weight roofs, elements of machine structures etc. Their disadvantage is that, if the rib height is smaller than approx. 800 mm, the face plates cannot be welded to ribs from inside. Then these connections can be realized from outside by arc-spot-welding, electron beam welding, slot or plug welds.

A brief survey of selected literature is given in [1]. The research has been carried out predominantly in the field of double bottoms of ships. The present paper is a generalization of the investigation of the minimum cost design of square cellular plates [1]. Our aim is to show how to optimize the plate dimensions, mainly the numbers of ribs. It will be shown by numerical examples the effect of fabrication costs and the yield stress of steel on the optimal number of ribs.

2. The cost function

It is assumed that the fabrication has the following steps. First the grid of ribs is welded from cold-formed channels or from welded I-beams. The grid nodes should be completely welded to be able to carry bending moments and shear forces. Then the elements of the upper and lower cover plates are welded to the ribs from outside with fillet welds (Fig.1.). This method is selected since one cannot find cost data for other welding methods. The ribs are continuous in y direction and intermittent in x direction. The cross-sectional area of a rib is approx. $2ht_r$ where h is the height, t_r is the thickness. The integer numbers of rib distances are φ_x and φ_y, respectively (Fig.1.) .The number of ribs in x-and y direction is φ_x+1 and φ_y+1, resp. Assuming that all ribs have the same cross-sectional area, the whole volume of the cellular plate is

$$V = 2 b_x b_y t_f + 2b_x h t_{rx} (\varphi_y+1) + 2b_y h t_{ry} (\varphi_x+1) \quad (1)$$

where t_f is the thickness of face plates. The total cost consists of the material and fabrication costs

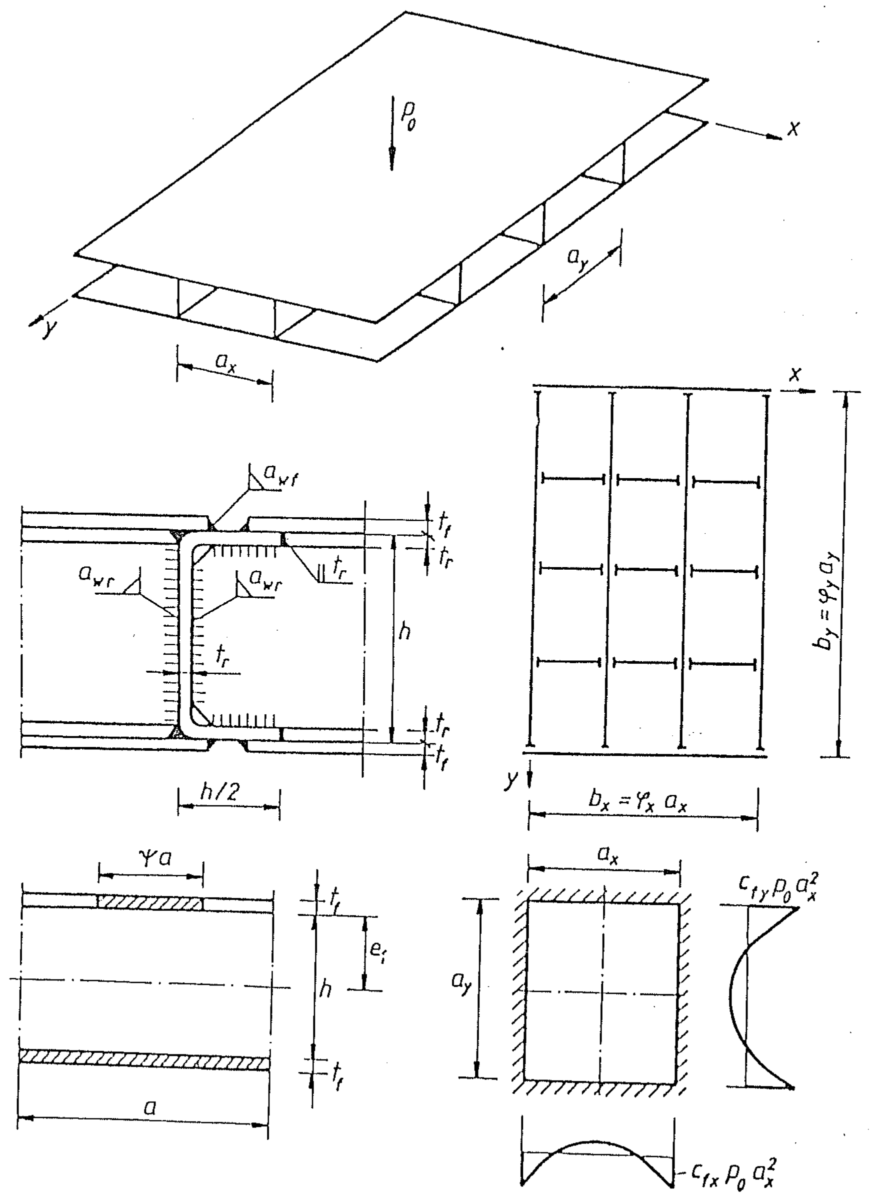

Fig.1. Details of a welded rectangular cellular plate

$$K = K_m + K_f = k_m \rho V + k_f \Sigma T_i \quad (2)$$

or in other form $\quad K/k_m = \rho V + k_f/k_m (T_1 + T_2 + T_3) \quad (3)$

where k_m(\$/kg) and k_f (\$/min) are the material and fabrication cost factors, resp, ρ is the material density, T_i are the fabrication times in min.

In order to give internationally usable solutions, the following ranges of k_m and k_f may be considered. For steel Fe 360 k_m = 0.5-1.2 \$/kg, for fabrication including overheads k_f = 15-45 \$/manhour = 0.25-0.75 \$/min. Thus the ratio k_f/k_m may vary in the range 0 - 1.5 kg/min. The value $k_f/k_m = 0$ in (3) corresponds to the minimum volume design. For the calculation of times T_i we use the method proposed by Pahl and Beelich [2].

a./ Preparation, assembly and tacking

$$T_1 = C_1 \delta \sqrt{\rho V}\sqrt{\kappa}; \; C_1 = 1.0 \text{ min/kg}^{0.5} \quad (4)$$

where δ is a difficulty factor, κ is the number of structural elements to be assembled. Number of continuous ribs in y direction is φ_x+1, number of rib elements in x direction (internal intermittent, peripheral continuous, Fig.1) is $\varphi_x(\varphi_y-1)+2$, number of face plate elements is $2\varphi_x \varphi_y$. Thus the number of all structural elements is $\kappa = 3(\varphi_x\varphi_y + 1)$ and $T_1 = 3\sqrt{\rho V}\sqrt{3(\varphi_x \varphi_y + 1)}$ (5)

b./ Welding

$$T_2 = \Sigma C_{2i} \, a_{wi}^{1.5} L_{wi} \quad (6)$$

$C_2' = 0.8\times10^{-3}$ min/(mm$^{1.5}$xmm) for manual arc-welding, $C_2'' = 0.5\times10^{-3}$ min/(mm$^{1.5}$xmm) for CO_2-welding, a_w and L_w are the size and length of welds in mm, resp.

It is assumed that the grid nodes are joined by manual-arc-welding with fillet welds and the face plate elements are connected to the grid by CO_2-welded fillet welds.

c./ Electrode changing, weld deslagging and chipping

$$T_3 = \sqrt{\delta}\, \Sigma C_{3i} \, a_{wi}^{1.5} L_{wi}; \; C_3' = C_2'; \; C_3'' = C_2'' \quad (7)$$

The number of perpendicular joints of ribs is

$$2(\varphi_x +1) + 2\varphi_x(\varphi_y - 1) = 2(\varphi_x \varphi_y +1)$$

It is assumed that the webs of ribs are welded with fillet welds of size $a_w = 0.7t_{ry}$ in the length of 2h, and the flanges of ribs of length h are welded with welds of $a_w = t_{ry}$ (Fig.1). Thus the $T_2 + T_3$ times for manual-arc-welded nodes are (in min)

$$T_2' + T_3' = (1 + \sqrt{\delta})\, 0.8\times10^{-3} \times 2 (\varphi_x \varphi_y + 1) [h t_{ry}^{1.5} + 2h (0{,}7t_{ry})^{1.5}] \quad (8)$$

The total length of fillet welds for face plate elements is

$$2\varphi_x \varphi_y (2a_x + 2a_y) = 4\varphi_x \varphi_y (b_x/\varphi_x + b_y/\varphi_y)$$

and the fillet weld size is taken as $a_{wf} = 0.5\, t_f$, thus

$$T_2''+T_3'' = (1 + \sqrt{\delta})\, 0.5\times10^{-3} (0.5t_f)^{1.5} \times 4 \varphi_x \varphi_y (b_x/\varphi_x + b_y/\varphi_y), \quad (9)$$

all sizes in mm.

3. The design constraints

a./ Constraints on compressive elastic stresses in the central upper face plate element are as follows

$$\sigma_{x.max} + \sigma_{xf.max} \leq \sigma_{adm} \quad (10)$$

$$\sigma_{y.max} + \sigma_{yf.max} \leq \sigma_{adm} \qquad (11)$$

where σ_{adm} is the admissible stress, $\sigma_{x.max}$ and $\sigma_{y.max}$ are caused by the bending of the whole plate, $\sigma_{xf.max}$ and $\sigma_{yf.max}$ are normal stresses due to the local bending of the face plate element.

It can be verified, similarly to the case of a square cellular plate [3] that, because of the large torsional stiffness of cells, the whole rectangular cellular plate can be calculated as an isotropic one.

$$\sigma_{x.max} = M_{xmax} E_1 e_1 / B; \qquad \sigma_{y.max} = M_{ymax} E_1 e_1 / B \qquad (12)$$

According to the isotropic plate theory [4]

$$M_{xmax} = c_{mx}\, p\, b_x^2 \quad \text{and} \quad M_{ymax} = c_{my}\, p\, b_x^2$$

where $p = 1.1p_0$, with the factor of 1.1 the self weight is considered, c_{mx} and c_{my} are given in [4] for simply supported edges. The bending stiffness B is calculated considering the effective width of the compressed face plate element (Fig.1)

$$B = E_1 I / a = E_1 h^2 t_f \psi / (1+\psi); \quad e_1 = h/(1+\psi); \quad E_1 = E/(1-\nu^2) \qquad (13)$$

where E is the modulus of elasticity and ν is the Poisson's ratio.

We use here the effective width formula proposed by Usami and Fukumoto [5]

$$\psi = 0.75/\lambda_p; \quad \lambda_p = \frac{a}{t_f}\sqrt{\frac{12(1-\nu^2)\sigma_{max}}{4\pi^2 E}} \quad \text{or} \quad \psi = 1.426/\lambda_p; \quad \lambda_p = \frac{a}{t_f}\sqrt{\frac{\sigma_{max}}{E}} \qquad (14)$$

Substitution of (13) into (12) yields

$$\sigma_{x.max} = c_{mx}\, p\, b_x^2 / (t_f h \psi_y), \quad \text{and} \quad \sigma_{ymax} = c_{my} p\, b_x^2 / (t_f h \psi_x) \qquad (15)$$

Elimination of ψ from (15) is performed using (14) and (15) and yields

$$\sigma_{x.max} = c^2_{mx}\, p^2 b_x^4 a_y^2 / (1.426^2 t_f^4 E h^2); \quad \sigma_{ymax} = c^2_{my}\, p^2 b_x^4 a_x^2 / (1.426^2 t_f^4 E h^2) \qquad (16)$$

Furthermore $\sigma_{xf.max} = 6\, c_{fx}\, p_0\, a_x^2 / t_f^2$, and $\sigma_{yf.max} = 6\, c_{fy}\, p_0\, a_x^2 / t_f^2$, (17)

where c_{fx} and c_{fy} are given in [4] for a uniformly loaded rectangular isotropic plate with clamped edges. Since c_{fx} and c_{fy} vary during the optimization procedure, these values are calculated with approximate analytical formulae in the form of a polynom

$$c_{fx} = c_0 + c_1 a_y/a_x + c_2 (a_y/a_x)^2 + c_3 (a_y/a_x)^3,$$

On the contrary, values of c_{mx}, c_{my}, c_{qx} and c_{qy} are constant during an optimization procedure, because b_y and b_x are given in a numerical example.

b./ Constraints on local buckling of rib webs due to bending

$$\sigma_{x.max} \leq \frac{23.9\pi^2 E_1}{12\gamma_b}\left(\frac{t_{rx}}{h}\right)^2 \quad \text{and} \quad \sigma_{y.max} \leq \frac{23.9\pi^2 E_1}{12\gamma_b}\left(\frac{t_{ry}}{h}\right)^2 \qquad (18\text{ a,b})$$

where γ_b is the safety factor for buckling.

c./ Constraints on local buckling of rib webs due to shear

$$\tau_x = \frac{Q_x a_y}{h t_{rx}} = \frac{c_{qx} p b_x a_y}{h t_{rx}} \le \frac{5.34\pi^2 E_1}{12\gamma_b}\left(\frac{t_{rx}}{h}\right)^2; \quad \tau_x \le \tau_{adm} \tag{19a}$$

$$\tau_y = \frac{Q_y a_x}{h t_{ry}} = \frac{c_{qy} p b_x a_x}{h t_{ry}} \le \frac{5.34\pi^2 E_1}{12\gamma_b}\left(\frac{t_{ry}}{h}\right)^2; \quad \tau_y \le \tau_{adm} \tag{19b}$$

where $\tau_{adm} = \sigma_{adm}/\sqrt{3}$ is the admissible shear stress, c_{qx} and c_{qy} are given in [4].

d./ Size constraints are the thickness limitations

$$t_{rx} \ge t_0; \quad t_{ry} \ge t_0 \quad \text{and} \quad t_f \ge t_0 \tag{20}$$

where t_0 is the minimal thickness considering the welding technology. Note that the deflection constraint is not considered here because of the large stiffness of the whole cellular plate.

4. The optimization procedure

In a numerical example the values of p_0, b_y, b_x, σ_{adm}, E, υ, c_{mx}, c_{my}, c_{qx}, c_{qy}, t_0 are given, and the unknowns to be optimized for minimum cost K_{min} are as follows: φ_x, φ_y, h, t_f, t_{rx} and t_{ry}. In the cost function the k_f/k_m ratio is varied in the range of 0 - 1.5. For the purpose of comparison we have used here three mathematical programming methods.

a./ The "Backtrack" combinatorial method is advantageous here, since the number of variables is only 6, φ_x and φ_y are integer numbers and the thicknesses should be commercially available, so the series of discrete values to be investigated can easily be defined. The starting point should be feasible. The detailed description of Backtrack with more numerical applications can be found in [6].

b./ The "Hillclimb" method is proposed by Rosenbrock. The method of rotating coordinates is a further development of the Hooke and Jeeves method. No derivatives are required. The starting point should be feasible. We have supplemented this method with a secondary search for finding discrete values after having continuous ones [7].

c./ FSQP - feasible sequential quadratic programming - method. CFSQP 1.0 is a set of C subroutines for the minimization of smooth objective functions subject to general smooth constraints [8]. If the initial guess provided by the user is infeasible for some constraints, CFSQP first generates a feasible point. Nonlinear equality constraints are turned into inequality constraints. The user must provide subroutines that define the objective functions and constraint functions or require that CFSQP estimates them by forward finite differences. CFSQP solves a modified optimization problem with only linear constraints and nonlinear inequality constraints. An Armijo-type line search is used to generate an initial feasible point when required. After obtaining feasibility, either an Armijo-type line search may be used or a nonmonotone line search is made and analyzed. The C version is a quite new development and we have worked also with the beta version on PC.

All of the programs are written in C and run under Borland C++ on PC 486 type computer. These codes are quicker than the Fortran and Basic codes and are more transportable, we also could run them on workstation.

5. Numerical examples

Data: the intensity of the uniformly distributed normal load $p_0=5\times10^{-3}$ N/mm^2, $p = 1.1\ p_0=5.5\times10^{-3}$ N/mm^2, $t_0 = 2$ mm, $E = 2.1\times10^5$ MPa, $\upsilon=0.3$, $\rho= 7850$ kg/m^3, $\gamma_b= 2$, $\delta=3$, $b_x = 10$ m. To show the effect of yield stress of steel, calculations are made for steel Fe 360 with $\sigma_{adm}= 120$ MPa, and for steel Fe 510 with $\sigma_{adm}= 120\times355/235 = 181$ MPa.

To show the effect of fabrication costs calculations are made for $k_f/k_m = 0;\ 0.5;\ 1.0$ and 1.5. The results are shown in Figs 2-3 and Table 1. Fig.2 shows the curves of the objective function as a function of φ_x in the vicinity of the optimum value. It can be seen that for larger k_f/k_m values - larger fabrication costs - $\varphi_{x\ opt}$ is smaller. With the use of higher-strength steel Fe 510 4-12 % cost savings can be achieved. The sensitivity of the objective function is small. In Fig.3 the minimal K/k_m cost values are plotted in function of b_y/b_x for steels Fe 360 and Fe 510. It can be seen that the K/k_m -values vary with b_y/b_x approximately linearly. These results are obtained by Backtrack programming method.

Table 1. Results of a numerical example with $b_x = 10$ m, $b_y = 14$ m, steel Fe 360 obtained by three mathematical programming methods, dimensions in mm

Method	k_f/k_m	φ_x	φ_y	t_f	t_{rx}	t_{ry}	h	K/k_m (kg)
CFSQP without discre-tization	0	13.0	16.0	5.1	2.2.	2.0	278	14548
	0.5	12.8	16.0	5.1	2.2	2.0	285	23627
	1.0	10.2	8.8	6.1	3.0	2.4	383	31743
	1.5	9.4	8.0	6.0	3.1	2.5	406	38705
Hillclimb without discre-tization	0	13.1	15.7	5.0	2.3	2.0	299	14624
	0.5	10.0	9.0	6.0	3.0	2.4	400	24609
	1.0	10.0	8.0	6.0	3.2	2.5	499	31888
	1.5	8.0	7.9	7.0	3.1	2.6	374	39094
Hillclimb with discre-tization	0	14	15	5	3	2	300	15229
	0.5	10	9	6	3	3	400	25459
	1.0	10	8	6	4	3	450	33485
	1.5	8	8	7	4	3	375	40665
Backtrack	0	13	16	5	3	3	300	16162
	0.5	10	9	6	4	3	400	26149
	1.0	10	8	6	4	3	450	33485
	1.5	8	8	7	4	3	375	40665

In Table 1. the optimal dimensions obtained by three methods are given for a numerical example. It can be seen that the Hillclimb and CFSQP methods resulted in very similar undiscretized optimal values. The results obtained by Hillclimb with discretization and by the discrete Backtrack are also very similar.

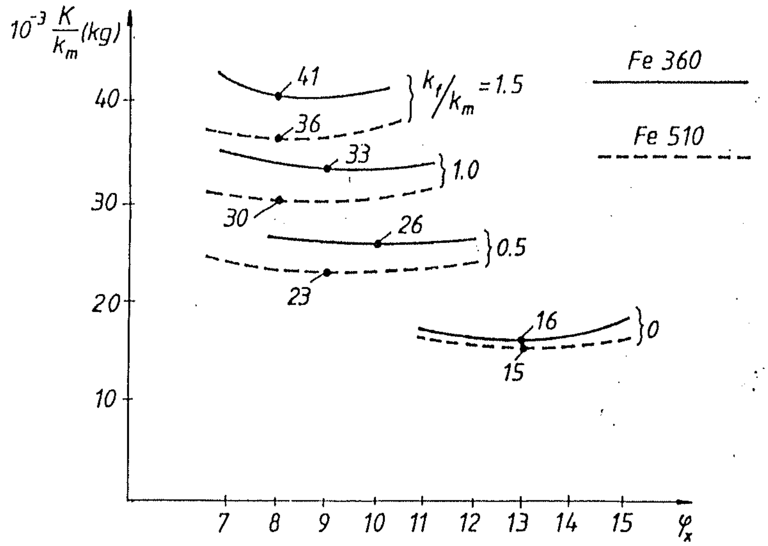

Fig.2. Results of the numerical example: minimal costs for various k_f/k_m-ratios and the $\varphi_{x.opt}$ values

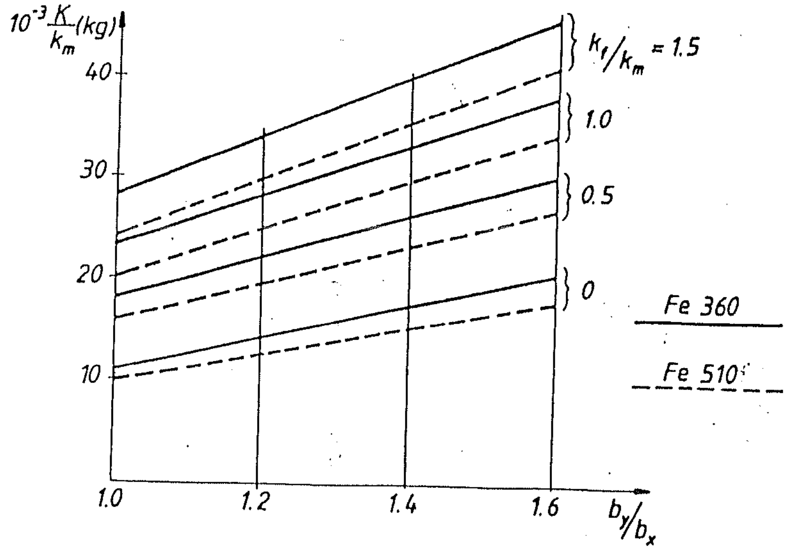

Fig.3. Results of the numerical example: minimal costs for various k_f/k_m-ratios in function of b_y/b_x

6. Conclusions

Illustrative numerical examples show that, because of the large torsional stiffness of cellular plates, relatively large structures can be realized using thin plates. The optimal number of ribs decreases when the fabrication cost k_f/k_m increases. The sensitivity of the objective function is small. The use of Fe 510 instead of Fe 360 results in 4-12 % cost savings. Active constraints are the normal stress limitation (10) and the constraints on local shear buckling of rib webs (19).

The comparison of the three mathematical programming methods shows that the Hillclimb technique is quick but can result in local minima, the Backtrack is suitable for few variables defined by series of discrete values, the CFSQP method is very robust and the starting point can be infeasible.

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