DESIGN AND ANALYSIS OF WELDED STRUCTURES

Minimum cost design of an orthogonally stiffened welded steel plate with a deflection constraint

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Abstract: An assembly desk is constructed as a square plate stiffened by an orthogonal grid of ribs. The residual welding deflection is calculated applying the Okerblom's method. When the ribs are tacked to each other and to the base plate before welding, then the deflection is decreased by grid effect. The base plate thickness and the dimensions of stiffeners are optimized to minimize the cost and to fulfil the deflection constraint.. The optimization is performed with and without grid effect and it is shown that the grid effect decreases the cost significantly.

Introduction

The present study deals with the design of an assembly desk, for which the deflection constraint assures the exact operation, fitness for assembly and fabrication of structural parts. The sufficient stiffness is guaranteed by using a welded stiffened plate construction. The shrinkage of eccentric welds connecting the stiffeners to the base plate causes deflections, which should be considered in the desk design.

In the case of a square desk an orthogonal stiffening is used. The main aim of this study is to show how to calculate the residual welding deflections in the case of an orthogonally stiffened plate. We have adapted the Okerblom's calculation method worked out for longitudinal welds of a single straight beam. We apply this method for the case of orthogonal stiffenings.

An orthogonally stiffened plate can be fabricated by two different welding sequences as follows: (a) welding of continuous stiffeners in one direction to the base plate with a cost effective welding method (SAW), then welding the interrupted stiffeners in other direction using GMAW for longitudinal welds and SMAW for nodes of connecting stiffeners, (b) the whole stiffened plate is assembled by tacking of stiffeners to the base plate and to each other, then welding of longitudinal welds by GMAW and node welds by SMAW.

Since in the method (b) the nodes can transfer the bending moments, the residual deflections can be calculated as a grid structure. The Okerblom's method is used for a grid structure. It is shown that the grid-effect decreases the deflections significantly. In the case of open section stiffeners the torsionless grid calculation method is used.

The cost function for both welding sequences are formulated and minimized searching for optimum base plate thickness and stiffener dimensions, while the number of stiffeners is fixed. The more advantageous welding sequence is determined by cost comparison.

Residual welding deflection from longitudinal welds of a straight beam

The books [1,2,3,4] give suitable calculation methods. The Okerblom's method gives relatively simple formulae, so it is adapted and applied [5,6,7].

In this method it is assumed that (a) the coefficient of thermal expansion and the Young modulus are independent from the temperature, (b) the deflections are in the elastic range, the Hooke-law is valid, (c) the cross sections of the beam will be planar after deflection, (d) the cross section is uniform, (e) the beam is made of one material grade, (f) the thermal distribution is uniform along the length of the beam and steady state.

The thermal shrinkage impulse A_T, which causes the residual stresses and deformations in the structure can be calculated as

$$A_T = \frac{0.4840\alpha_o Q_T}{c_o \rho t}\ln 2 = \frac{0.3355\alpha_o Q_T}{c_o \rho t} \qquad (1)$$

where $Q_T = \eta_o \frac{UI_w}{v_w} = q_o A_w$, U arc voltage, I arc current, v_w speed of welding, c_o specific heat, η_o coefficient of efficiency, q_0 is the specific heat for the unit welded joint area (J/mm^3), A_w is the welded joint area. It can be seen that this formula contents all the important material and welding parameters. Thus, it can be used not only for steels but also for other materials, e.g. for aluminium-alloys.

For a mild or low alloy steels, where α_o=12x10^{-6} [1/C^o], $c_o\rho$= 4.77x10^{-3} [J/mm^3/C^o], the thermal impulse is

$$A_T t \ [\text{mm}^2] = 0.844x10^{-3} Q_T \ [\text{J/mm}] \qquad (2)$$

and the basic Okerblom formulae for the strain at the centre of gravity and the curvature are as follows

$$\varepsilon_G = \frac{A_T t}{A} = -0.844x10^{-3}\frac{Q_T}{A} \quad (3)$$

$$C = \frac{A_T t\, y_T}{I_x} = -0.844x10^{-3}\frac{Q_T y_T}{I_x} \quad (4)$$

The minus sign means shrinkage. Furthermore

$$Q_T = \eta_0 \frac{UI_w}{v_w} = \eta_0 \frac{3600U[V]\rho}{\alpha_N} A_w \quad (5)$$

is the thermal impulse due to welding, A cross-sectional area, I_x moment of inertia of the beam cross-section, A_w cross-sectional area of the weld, $\rho = 7.85x10^{-6}$ kg/m^3 is the density of steel, $\alpha_N = 8.8x10^{-3}$ kg/Ah (Amper-hour) is the coefficient of penetration..

With values of U = 27V, $\eta_0 = 0.7$ for butt welds

$$Q_T \text{ (J/mm)} = 60.7A_w \text{ (mm}^2\text{)}, \quad (6)$$

for SMAW (shielded metal arc welding) fillet welds

$$Q_T = 78.8a_w^2 \quad (7)$$

and for GMAW (gas metal arc welding) of SAW (submerged arc welding) fillet welds

$$Q_T = 59.5a_w^2. \quad (8)$$

a_w is the fillet weld size.

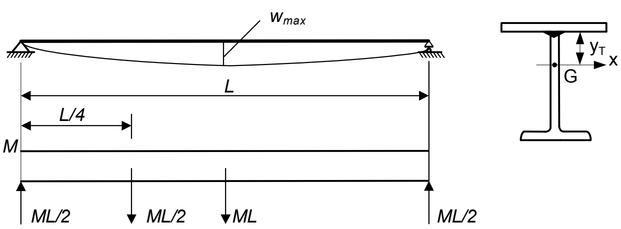

Figure 1. Calculation of the maximum deflection for a simply supported welded beam of constant cross-section

The maximum deflection due to shrinkage of a single eccentric longitudinal weld in the case of a simply supported beam of constant cross-section can be calculated using the correlation between the distributed load p, bending moment M and deflection w

$$\frac{d^2 M(z)}{dz^2} = p(z) \quad (9)$$

and

$$\frac{d^2 w(z)}{dz^2} = \frac{M(z)}{EI_x} = C \quad (10)$$

i.e. the deflection can be obtained by calculating the bending moment considering the bending moment diagram as a virtual loading (Figure 1)

$$w_{\max} = \frac{ML}{2EI_x}\cdot\frac{L}{4} = \frac{ML^2}{8EI_x} = \frac{CL^2}{8} \quad (11)$$

The longitudinal shortening is

$$\Delta L = \varepsilon_G L \quad (12)$$

which is important for fabrication to enable the assembly of structural components.

Residual welding curvatures in an orthogonally stiffened plate

Figure 2 shows the deformations of a plate stiffened by two perpendicular stiffeners. First, the stiffener 1 is welded and its point A moves to point B and the stiffeners became the form of 1' and 2'. Secondly, welding of stiffener 2 causes a further curvature and the stiffeners became the form of 1'' and 2''. Thus, the curvatures are added to each other. In the case of two stiffeners in a plate of square symmetry the curvatures double. It is also the case of more stiffeners of square symmetry without grid-effect.

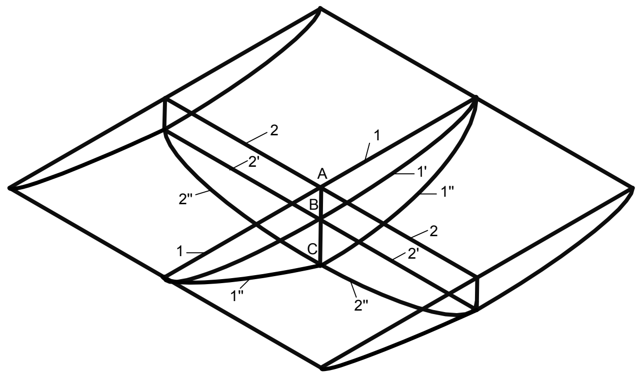

Figure 2. Deformations of a plate stiffened by two perpendicular stiffeners

The grid effect

This effect is illustrated by an example of a rectangular plate orthogonally stiffened by one-one stiffener (Figure 3). When the stiffeners are previously tacked to the base plate and to each other, the node can transfer the bending moments and the grid-effect acts. The unknown force X acting in the node A can be calculated using the force method, i.e. from a deflection equation expressing that the deflection of two stiffeners caused by welding curvature and by force X are identical.

Using the method described in Section 2, the deflection of the stiffener 1 from the welding curvature is

$$EI_1 w_{1M} = \frac{3Ma^2}{2} - \frac{Ma^2}{2} = Ma^2; w_{1M} = C_1 a^2 \quad (13)$$

and from the force X

$$w_{1X} = \frac{5Xa^3}{9} - \frac{Xa^3}{9} = \frac{4Xa^3}{9} \quad (14)$$

The deflections of the stiffener 2 are

$$w_{2M} = \frac{C_2(2a)^2}{8} = \frac{C_2 a^2}{2} \quad (15)$$

and

$$w_{2X} = \frac{Xa^2}{4}\frac{2a}{3} = \frac{Xa^3}{6} \tag{16}$$

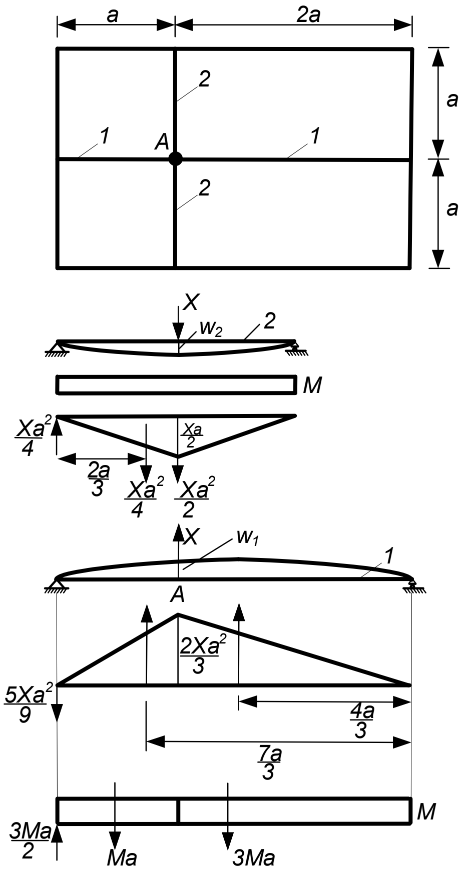

Figure 3. A plate orthogonally stiffened by one-one stiffener

The deflection equation can be expressed as

$$w_{1M} - w_{1X} = w_{2M} + w_{2X} \tag{17}$$

the unknown force from Eq (17) is

$$X = \frac{9(2C_1 - C_2)}{11} \tag{18}$$

and the deflection of point A considering the grid-effect

$$w_A = w_{2M} + w_{2X} = \frac{C_2 a^2}{2} + \frac{3(2C_1 - C_2)a^2}{22} = \frac{(3C_1 + 4C_2)a^2}{11} \tag{19}$$

and without the grid-effect

$$w_{A0} = w_{1M} + w_{2M} = (C_1 + 0.5C_2)a^2 \tag{20}$$

If $C_1 = C_2 = C$

$$w_A = 0.6363Ca^2 \tag{21}$$

and

$$w_{A0} = 1.5Ca^2 \tag{22}$$

i.e. the grid-effect decreases the deflection by 57%.

Assembly desk of square symmetry with 4-4 stiffeners

We assume that the stiffeners are previously tacked to the base plate and to each other, the grid-effect acts. In the nodes of 1, 4, 6 and 7 in the Figure 4. forces do not act because of symmetry, in the others the same force *X* acts. This force can be determined using a deflection equation. This equation expresses that, for example, the deflection of node 2 caused by the shrinkage and force *X* from the stiffener 1-1' and 2-8 is the same.

Solution of the gridwork from shrinkage of welds (Figure 4)

Deflection of the stiffener 1-1' at the node 2 from the shrinkage is

$$EIw_{2M}(1-1') = \frac{ML}{2}\frac{2L}{5} - \frac{2ML}{5}\frac{L}{5} = \frac{3ML^2}{25};$$

$$w_{2M}(1-1') = \frac{3CL^2}{25} \tag{23}$$

and from the forces *X*

$$w_{2X}(1-1') = \frac{3XL^2}{25}\frac{2L}{5} - \frac{2XL^2}{25}\frac{2L}{15} = \frac{14XL^3}{375} \tag{24}$$

Deflection of the stiffener 2-8 at the node 2 from the shrinkage

$$EIw_{2M}(2-8) = \frac{ML}{2}\frac{L}{5} - \frac{ML}{5}\frac{L}{10} = \frac{2ML^2}{25};$$

$$w_{2M}(2-8) = \frac{2CL^2}{25} \tag{25}$$

and from forces *X*

$$w_{2X}(2-8) = \frac{2XL^2}{25}\frac{L}{5} - \frac{XL^2}{50}\frac{L}{15} = \frac{11XL^3}{750} \tag{26}$$

The deflection equation can be expressed as

$$w_{2M}(1-1') - w_{2X}(1-1') = w_{2M}(2-8) + w_{2X}(2-8) \tag{27}$$

The unknown force *X* from Eq. (26)

$$X = \frac{10C}{13L} \tag{28}$$

The maximum deflection at the node 4 considering the grid-effect is

$$w_4 = w_{2M}(1-1') + w_{4X}(2-8) = \frac{3CL^2}{25} + \frac{17XL^3}{750} = \frac{134CL^2}{975} = 0.1379CL^2 \tag{29}$$

The deflection without the grid-effect, according to the statement detailed in Section 3, is calculated as double of the deflection of a stiffener

$$w_{40} = \frac{6CL^2}{25} = 0.24CL^2 \tag{30}$$

i.e. the grid-effect decreases the deflection by 42%.

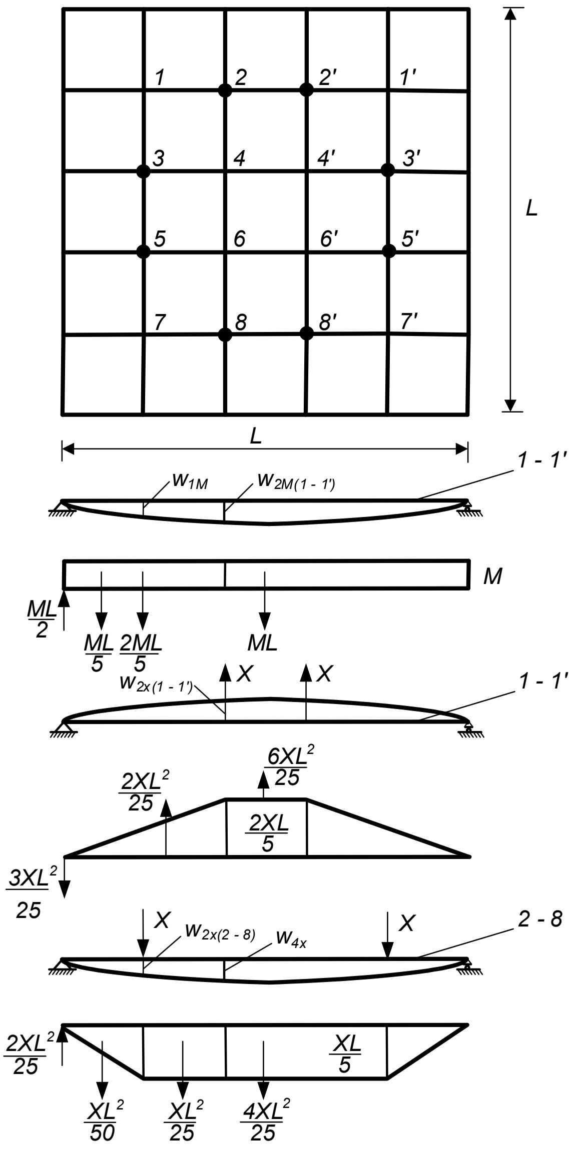

Figure 4. A plate orthogonally stiffened by 4-4 stiffeners

Solution of the gridwork from the uniformly distributed normal load (Figure 5)

Since this load acts after the fabrication, the calculation considers the grid effect.

The deflection equation for the unknown force X_p is the same as in Section 5.1.

$$w_{2p}(1-1') - w_{2Xp}(1-1') = w_{2p}(2-8) + w_{2Xp}(2-8) \quad (31)$$

The bending moments in Figure 5 are as follows:

$$M_1 = pa^3; M_2 = \frac{3}{2}pa^3 \quad (32)$$

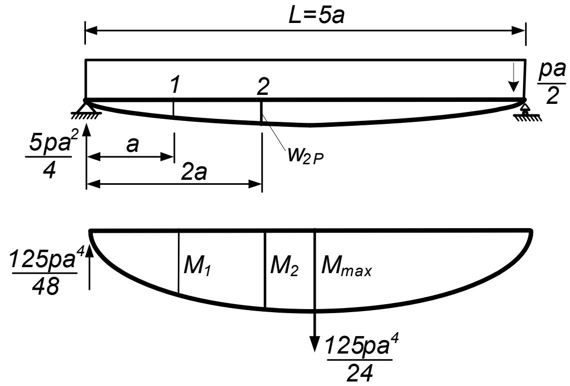

Figure 5. Bending moments from uniformly distributed normal load

The corresponding deflections are expressed as

$$EI_{xe}w_{2p}(1-1') = \frac{125}{24}pa^5 - \frac{pa^5}{3} - pa^5 = \frac{31}{8}pa^5 \quad (33)$$

$$EI_{xe}w_{2p}(2-8) = \frac{125}{48}pa^5 - \frac{pa^5}{48} - \frac{pa^5}{6} = \frac{29}{24}pa^5 \quad (34)$$

$$EI_{xe}w_{xp}(1-1') = \frac{14}{3}X_p a^3 \quad (35)$$

$$EI_{xe}w_{2xp}(2-8) = \frac{11}{6}X_p a^3 \quad (36)$$

From the deflection equation one obtains

$$X_p = \frac{16}{39}pa^2 = \frac{16}{975}pL^2 \quad (37)$$

The maximum bending moment at the middle of nodes 4 and 6:

$$M_{\max} = \frac{25pa^3}{16} + X_p a = \frac{1231}{624}pa^3 = 1.9728pa^3 \quad (38)$$

The maximum deflection at the node 4 is

$$w_4 = w_{2p}(1-1') + w_{4xp}(2-8) = \frac{31p_1a^5}{8EI_x} + \frac{17x16p_1a^5}{6x39EI_x} = \frac{5.0374p_1a^5}{EI_x} \quad (39)$$

where the intensity of the normal load without safety factors is

$$p_1 = p_0 + \rho_1\frac{V}{L^2} \quad (40)$$

and the moment of inertia I_x is given in Section 6.2.

Minimum cost design of the assembly desk with 4-4 stiffeners considering the grid-effect

A lot of the optimum design problems relating to stiffened plates have been worked out in the books of Farkas and Jármai [8,9].

Numerical data: L = 6000, a = 1200 mm, p_0 = 5000 N/m^2 = 5x10^{-3} N/mm^2, f_y = 235, f_{y1} = f_y/1.1, E = 2.1x10^5 MPa

Stress constraint

The factored intensity of the uniformly distributed normal load considering also the self mass

$$p = 1.5p_0 + 1.1\rho_1 \frac{V}{L^2}, \quad V = L^2 t + 8Lht_w \tag{41}$$

$$\sigma_{max} = \frac{M_{max}}{W_{xe}} \le f_{y1} \tag{42}$$

$$W_{xe} = \frac{I_{xe}}{y_{Ge}} \tag{43}$$

Cross-section area of a stiffener (Figure 6)

$$A = ht_w + at \tag{44}$$

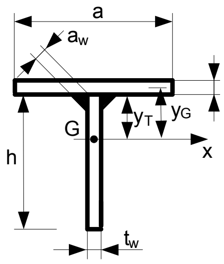

Figure 6. Dimensions of a stiffener

The effective width of the base plate according to the DNV design rules

$$a_e = \frac{a(\lambda_p - 0.22)}{\lambda_p^2}, \quad \lambda_p = 0.525\frac{a}{t}\sqrt{\frac{f_y}{E}} \tag{45}$$

Effective cross-section area is

$$A_e = ht_w + a_e t \tag{46}$$

The moment of inertia of the effective stiffener cross-section

$$I_{xe} = \frac{a_e t^3}{12} + a_e t y_{Ge}^2 + \frac{h^3 t_w}{12} + ht_w\left(\frac{h+t}{2} - y_{Ge}\right)^2 \tag{47}$$

where the distance of the gravity centre is

$$y_{Ge} = \frac{ht_w}{A_e}\frac{h+t}{2} \tag{48}$$

Deflection constraint

$$w_{max} = 5.0374\frac{p_{10}a^5}{EI_x} + 0.1379CL^2 \le w_{adm} = \frac{L}{1000} \tag{49}$$

where

$$p_{10} = p_0 + \frac{\rho_1 V}{L^2} \tag{50}$$

$$I_x = \frac{at^3}{12} + aty_G^2 + \frac{h^3 t_w}{12} + ht_w\left(\frac{h+t}{2} - y_G\right)^2 \tag{51}$$

$$y_G = \frac{ht_w}{A}\frac{h+t}{2} \tag{52}$$

$$C = 0.844x10^{-3}\frac{Q_T y_T}{I_x} \tag{53}$$

$$Q_T = 59.5a_w^2, \quad a_w = 0.4t_w, a_{w\min} = 3 \text{ mm} \tag{54}$$

$$y_T = y_G - \frac{t}{2} \tag{55}$$

Cost function

Welding of the base plate with 3 butt welds from 4 strips (SAW) assuming that the plate thickness t>15 mm:

$$K_{w1} = k_w\left(\Theta_1\sqrt{4\rho V_1} + 1.3x0.1033x10^{-3}t^{1.9}3L\right) \tag{56}$$

where $k_w = 1$ \$/min, $\Theta_1 = 2$, $V_1 = L^2 t$

Welding of the continuous and intermittent stiffeners to the base plate and the intermittent stiffeners to the continuous ones in 16 nodes:

$$K_{w4} = k_w\left(\Theta_2\sqrt{25\rho V_3} + 1.3x0.3394x10^{-3}a_w^2 16L + T_1\right) \tag{57}$$

$$T_1 = 1.3x0.7889x10^{-3}a_w^2 16x4h, \quad \Theta_2 = 3 \tag{58}$$

$$V_3 = V_1 + 8Lht_w \tag{59}$$

Painting cost

$$K_P = k_P S, \quad S = 2L^2 + 16Lh \tag{60}$$

$k_P = 28.8\text{x}10^{-6}$ \$/mm^2

Material cost

$$K_M = k_M \rho V_3, k_M = 1\$/kg \tag{61}$$

Total cost

$$K = K_M + K_{w1} + K_{w4} + K_P \tag{62}$$

Results of optimization

Optimization is performed using a MathCad algorithm. Results are summarized in Table 1.

TABLE 1. RESULTS OF OPTIMIZATION CONSIDERING THE GRID EFFECT. DIMENSIONS AND DEFLECTIONS IN MM. THE ADMISSIBLE DEFLECTION IS 6 MM.

h	t_w	t	w_{max}	K (\$)
240	18	23	5.95	16490
250	18	21	5.87	15850
260	**19**	**19**	**5.95**	**15670**
270	20	18	5.96	15920
280	20	17	5.88	15700
290	21	17	5.77	16370

The optimum is marked by bold letters. The stress constraint is passive, in the case of the optimum solution (h = 260 mm) σ_{max} = 80 < 213.6 MPa.

Minimum cost design of the assembly desk without grid effect

The constraints on stress and deflection are formulated similarly to the previous section, but in the deflection constrained Eq. (49) the second member 0.1379 CL^2 is replaced by 0.24 CL^2 according to Eq. (30).

Cost function

The subscript 1 in the cost function relates to the desk without grid effect. Welding of the base plate with 3 butt welds from 4 strips (SAW) assuming that the plate thickness t_1>15 mm:

$$K_{w11} = k_w\left(\Theta_1\sqrt{4\rho V_{11}} + 1.3x0.1033x10^{-3}t_1^{1.9}3L\right) \quad (63)$$

where k_w = 1 \$/min, $\Theta_1 = 2$, $V_{11} = L^2 t_1$

Welding of four continuous stiffeners with double fillet welds (SAW)

$$K_{w2} = k_w\left(\Theta_1\sqrt{5\rho V_2} + 1.3x0.2349x10^{-3}a_1^2 8L\right) \quad (64)$$

where

$$V_2 = V_{11} + 4Lh_1t_{w1} \quad (65)$$

Welding of the intermittent stiffeners with double fillet welds to the base plate and to the continuous stiffeners (GMAW-C)

$$K_{w3} = k_w\left(\Theta_2\sqrt{21\rho V_{31}} + 1.3x0.3394x10^{-3}a_{w1}^2 8L + T_{11}\right) \quad (66)$$

$$T_{11} = 1.3x0.7889x10^{-3}a_{w1}^2 16x4h_1 \quad (67)$$

$$V_{31} = V_2 + 4Lh_1t_{w1} \quad (68)$$

Cost of painting

$$K_{P1} = k_P S_1, \quad S_1 = 2L^2 + 16Lh_1 \quad (69)$$

Material cost

$$K_{M1} = k_M\rho V_{31} \quad (70)$$

Total cost

$$K_1 = K_{M1} + K_{w11} + K_{w2} + K_{w3} + K_{P1} \quad (71)$$

Results of optimization

Optimization is performed using a MathCad algorithm. Results are summarized in Table 2.

The optimum is marked by bold letters. The stress constraint is passive, in the case of the optimum solution (h = 250 mm) σ_{max} = 90 < 213.6 MPa.

Conclusions

The calculation method of residual welding deflection worked out for simple beams with longitudinal eccentric welds can be applied for orthogonally stiffened plates as well.

TABLE 2. RESULTS OF OPTIMIZATION WITHOUT THE GRID EFFECT. DIMENSIONS AND DEFLECTIONS IN MM. THE ADMISSIBLE DEFLECTION IS 6 MM.

h_1	t_{w1}	t_1	w_{max1}	K_1 (\$)
240	18	26	5.795	18570
245	18	25	5.88	18160
250	**18**	**24**	**5.98**	**17750**
255	19	24	5.94	18230
260	19	24	5.80	18310
270	20	23	6.00	18430
280	20	23	5.75	18580
290	21	23	5.77	19210

In the case of a plate of square symmetry orthogonally stiffened by 4-4 flat stiffeners, two fabrication sequences are investigated as follows: (a) welding of continuous welds in one direction and welding of intermittent welds in other direction, (b) assembly of the whole stiffened plate by tacking and then welding the intermittent welds. In the later sequence the grid effect decreases the residual welding deflection significantly.

It can be seen from Tables 1 and 2 that the fabrication with grid effect decreases the total cost by 100(17750-15670)/17750 = 12%.

Acknowledgement

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