A New Petrophysical Model for Describing the Pressure-dependent Acoustic Velocity in Rocks

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SUMMARY

The pressure dependence of the velocity and absorption coefficient of seismic/acoustic waves in rock is an extensively explored rock physical problem. Based on simple physical assumptions, a new petrophysical model is developed, through which a relationship between velocity and pressure is set up and explained. The material parameters of the model are determined by using laboratory measurements data and linearized inversion method.
Introduction

In recent years non-conventional reservoirs with extremely low porosity and permeability have come to
the front in hydrocarbon prospecting. Knowledge of petrophysical parameters (porosity, permeability)
is extremely important in pressurized, deep-seated and high-temperature hydrocarbon reservoirs.
Therefore, it is important to create a reliable and reusable reservoir geology model. Acoustic
petrophysical characteristics (mostly the propagation velocity of elastic wave and its absorption
coefficient) and their pressure dependence play an important role in creating the petrophysical model
because of their straight connection to the porosity.

In the paper based on acoustic velocity measurement data made on core samples originated from some
domestic oil drilling wells, we introduce a new petrophysical model, through which a relationship
between velocity and pressure is set up and explained. The petrophysical parameters of the model are
determined by means of laboratory measurements.

Measurement of the acoustic wave velocity

The measurements necessitate the compilation of special measuring equipment. Knowing the sample
length, the wave propagation time is necessary to be determined which can be measured by
oscilloscope. Figure 1 shows the measurement layout.

![Figure 1 Measurement layout.](image)

Rock samples subjected to uniaxial stress were analyzed with an electromechanical pressing device.
The load was increased in small steps (~1000N) meanwhile the propagation time was measured at the
adjoined pressure. Measurements were carried out on 27 different rock samples including two typical
test results, which are presented in the paper.

The pressure dependent velocity model

The velocity of elastic wave in rocks depends on the type of rock made up of minerals and cementing
material (rock matrix), porosity of rock, pore-filling fluid as well as microcracking. Several qualitative
ideas exist to describe pressure dependence of the velocity. One such idea suggested by Birch (1960),
states that pore volume reduces with increasing pressure, thus increasing velocity can be measured.
Brace and Walsh (1964) explained the phenomenon of pressure dependence by closing of microcracks
at increasing pressure. Microcracks in the sample raised of rock body (2000-3000m depth) open with
ceased rock stress so probably less propagation velocity can be measured at atmospheric pressure in
laboratory than at original state of rock, in-situ. When rock samples are loaded by pressure,
microcracks are closed again which leads to a velocity increase. This process continues apparently until
all microcracks are closed. Accepting this qualitative idea we can create a new petrophysical model
based on the following formulation.

If we create $d\sigma$ stress increase in the rock, we will assume that $dN$ (the change of the number of open
microcracks) is directly proportional to the $d\sigma$ stress increase and to the $N$ number of open
microcracks. At the same time if there are more microcracks in the rock, then there will be more
closing ones, too. We can describe the two assumptions with the following differential equation

$$dN = -\lambda N d\sigma \rightarrow N = N_0 \exp(-\lambda\sigma),$$  \hspace{1cm} (1)

where $\lambda$ is the proportionality factor (material quality dependent constant), $N$ is the number of open
microcracks and $N_0$ is the number of the open microcracks at stress-free state ($\sigma = 0$). The negative
sign represents that at increasing stress with closing microcracks the number of the open microcracks decreases.

Another assumption is the linear relationship between the $dv$ propagation velocity change due to $d\sigma$ pressure increment and $dN$ the number of closing microcracks. Combining this assumption with Eq.(1) we obtain

$$dv = -\alpha dN \rightarrow dv = \alpha N_0 \exp(-\lambda \sigma) d\sigma,$$

(2)

where $\alpha$ is the proportionality factor (another material quality dependent constant). The negative sign represents that the velocity is increasing with decreasing number of cracks. After integrating Eq.(2) we have

$$v = K - \alpha N_0 \exp(-\lambda \sigma).$$

(3)

In the unloaded state ($\sigma=0$) the propagation velocity can be measured. Its value is denoted by $v_0$ and can be computed from Eq.(3) as $v_0 = K - \alpha N_0$. From here we get the integration constant: $K = v_0 + \alpha N_0$. With this result and introducing the notation $\Delta v = \alpha N_0$ Eq.(3) can be rewritten in the following form

$$v = v_0 + \alpha N_0 (1 - \exp(-\lambda \sigma)) \rightarrow v = v_0 + \Delta v (1 - \exp(-\lambda \sigma)),$$

(4)

where $v_0$ is the velocity at which the elastic wave propagates in the stress-free rock and $\lambda = 1/\sigma^*$, where $\sigma^*$ is the stress value at which velocity (starting from $v_0$) approximates the $v_{\text{max}}$ value with $1/e$ accuracy.

Eq.(4) provides a theoretical connection between the propagation velocity and rock pressure. It means that velocity as a function of stress starts from $v_0$ and increases up to the $v_{\text{max}}=v_0+\Delta v$ value according to the function $1-\exp(-\lambda \sigma)$. Consequently the velocity reaches its $v_{\text{max}}$ limit at high stress values. The $\Delta v = v_{\text{max}}-v_0$ value is a velocity range in which the propagation velocity can vary from stress-free state up to the state characterized by high rock pressure. (Certainly it is only valid for the limiting case of the model, because in the range of high stresses new microcracks can arise in the rock. It can be mentioned that the description of the latter was not a goal in this approach. Samples were loaded during the measurements only up to one third of the uniaxial strength to avoid creating new cracks.)

**The determination of model parameters by inversion**

If propagation velocity at some stress value is known from measurements, then the parameters of the petrophysical model ($v_0$, $\Delta v$, $\lambda$) can be derived by using regression methods (Dobróka et al. 1991). We used linearized geophysical inversion method (the principle of the least squares method (Menke 1984)). Hence with the estimated parameters the propagation velocity can be determined at any pressure by means of our rock physical model equation.

**Laboratory experiments**

Inversion-based processing of loading data resulted in the following values of sample 1T2

$$v_0=2720 \text{ (m/s)}, \Delta v=988.2 \text{ (m/s)}, \lambda=0.09545 (\text{MPa})^{-1}$$

and those of sample 3T3 were

$$v_0=3496 \text{ (m/s)}, \Delta v=840 \text{ (m/s)}, \lambda=0.08366 (\text{MPa})^{-1}.$$
With the above given values velocities were calculated at any pressure substituting them to Eq.(4). The result is shown in Figure 2 and 3. The continuous line shows the calculated (by the least square principle) velocity-pressure function while symbols represent the measured data.

The figures show that the calculated curves are in good accordance with the measured data. The measure of fitting can be calculated by using the data misfit (D[%]) formula

\[ D = \sqrt{\frac{1}{N} \sum_{k=1}^{N} \left( \frac{d_k^{(m)} - d_k^{(c)}}{d_k^{(c)}} \right)^2} \cdot 100 \% , \]

where \( d_k^{(m)} \) is the measured velocity at the k-th pressure value and \( d_k^{(c)} \) is the k-th calculated velocity data, which can be computed according to Eq.(4). The value of data misfit was obtained 0,6\% for the sample 1T2 and 0,5\% for the sample 3T3.

**Petrophysical consequence**

The Wyllie formula (Wyllie et al. 1958) describes the connection between the porosity and the propagation velocity of elastic wave. This formula gives a good approximation for propagation velocity of elastic wave propagating in low porosity rocks

\[
\frac{1}{\nu} = \phi \frac{1}{v_1} + \frac{1 - \phi}{v_2} \quad \rightarrow \quad \phi = \frac{\frac{1}{v} - \frac{1}{v_2}}{\frac{1}{v_1} - \frac{1}{v_2}} \quad \rightarrow \quad \phi_0 = \frac{\frac{1}{v_0} - \frac{1}{v_2}}{\frac{1}{v_1} - \frac{1}{v_2}},
\]

where \( v_1 \) is in pore-filling fluid valid wave propagation velocity, \( v_2 \) is the velocity of rock matrix in monomineralic rocks or the average velocity of multimineral rock matrix, \( \phi \) is the porosity, \( \phi_0 \) is the porosity and \( v_0 \) is the measured propagation velocity at stress-free state (\( \sigma=0 \)). Combining this formula with our model, the expression gives the possibility to calculate the porosity at any pressure.
\[
\frac{\varphi}{\varphi_0} = \frac{1}{\nu_0 + \Delta \nu (1 - \exp(-\lambda \sigma))} - \frac{1}{\nu_2}.
\]

By the knowledge of the porosity measured at atmospheric pressure the porosity can be determined at any pressure (or depth). Figure 4 shows the typical behaviour curve of porosity rate vs. pressure. As it can be seen the porosity rate decreases with increasing pressure.

![Figure 4 Porosity rate-pressure function.](image)

Conclusions

The theoretical data calculated by inversion (using our developed model) match accurately with measured data proving that the petrophysical model applies well in practice. It was also applied on further 25 sandstone samples (fine-, medium-, coarse-grained, pebbly, tuffy etc.) with success during the research. The described inverse problem is significantly overdetermined, because the number of measurement data is much more than the number of parameters to be determined. Therefore, the inversion procedure is stable and can be handled as a linear inverse problem. Inversion results confirmed the accuracy and feasibility of our petrophysical model.

References


