16th

„Building Services, Mechanical and Building Industry Days"

International Conference
14-15 October 2010

DEBRECEN, HUNGARY

ISBN 978-963-473-423-9

16th

„Building Services, Mechanical and Building Industry Days"

International Conference
14-15 October 2010

DEBRECEN, HUNGARY

HEATING EFFECTS ON UNSTEADY LAMINAR FLOW PAST A CIRCULAR CYLINDER

Betti Bolló
Department of Fluid and Heat Engineering, University of Miskolc;
email: aramzb@uni-miskolc.hu

Keywords: *effective temperature, laminar flow, Strouhal number, Nusselt number, lift coefficient*

Abstract:

The two-dimensional flow around a stationary heated circular cylinder at low Reynolds numbers of $50 < Re < 210$ *is investigated numerically using the FLUENT commercial software package. The dimensionless vortex shedding frequency (St) reduces with increasing temperature at a given Reynolds number. The effective temperature concept was used and St-Re data were successfully transformed to the St-Re*$_{eff}$ *curve. Comparisons include root-mean-square values of the lift coefficient and Nusselt number. The results agree well with available data in the literature.*

1. Introduction

The incompressible flow past a stationary cylinder is a classical bluff body problem in fluid mechanics. Its physical and real-life applications have attracted the attention of engineers and scientists for over a century, and include situations such as the effect of wind on tall slender buildings, silos, and chimney stacks. Zdravkovich (1997) summarizes many theoretical and experimental investigations. For isothermal flow, the steady twin recirculation vortices behind a circular cylinder are susceptible to perturbations in the transverse direction and begin to shed downstream alternatively at a Reynolds number of approximately $\mathrm{Re}_c = 47$ (Norberg, 2001). Reynolds number Re is defined as $\mathrm{Re} = d u_\infty / \nu_\infty$, where u_∞ – is the free-stream velocity, [m/s], d – is the cylinder diameter, [m] and ν_∞ is the kinematic viscosity of the fluid at free stream temperature, [m^2/s]. At $\mathrm{Re} > 47$, the alternatively shed vortices develop into vortex rows, known as the von Kárman vortex street. The unsteady wake flow at this stage is also called laminar two-dimensional wake regime or laminar periodic wake regime.

For flows over a heated cylinder the fluid properties such as viscosity, density and thermal conductivity vary with the temperature. As a consequence, the thermodynamic properties in the system of governing equations also become temperature dependent. This has a significant effect on the flow characteristics and makes the flow phenomena much more complex than in the isothermal case. Lecordier et al. (1991), Dumouchel et al. (1998) and Wang et al. (2000) indicated that the vortex shedding in an air flow can be reduced or even completely suppressed by increasing the cylinder temperature. The temperature variation between the cylinder wall and the fluid leads to changes in the kinematic viscosity of fluid, so that the local Reynolds number varies in the near field of the heated cylinder. The critical Reynolds number Re_c increases linearly with the temperature ratio up to 2. The temperature ratio T^* is defined as the ratio of the cylinder wall temperature T_w and that of the free-stream value T_∞, i.e., $T^* = T_w / T_\infty$, [K]. This phenomenon leads to the development of the effective Reynolds number concept (Lecordier et al. (1991), Dumouchel et al. (1998), Wang et al., (2000)), which models the varying kinematic viscosity in the non-isothermal wake behind a heated circular cylinder by defining an effective temperature

$$T_{eff} = T_\infty + c\left(T_w - T_\infty\right) \quad (1)$$

where c is a coefficient. Different c values ranging from 0 to 1.0 have been chosen by different authors. The so-called "film" temperature T_f, defined as the arithmetic mean of the cylinder wall temperature and the free-stream temperature with the corresponding to the c value of 0.5 in Eq.(1), is the most frequently used artificial temperature in the literature to calculate the kinematic viscosity. This concept was first introduced by Lecordier et al. (1991), who originally suggested $c = 0.3$. Dumouchel et al. (1998) reported $c = 0.24$ and the correlation of Wang et al. (2000) is based on $c = 0.28$, which works well over the temperature ratio range of 1.1 to 1.8. A numerical study by Shi et al. (2004) concluded that for this coefficient ($c = 0.28$) their results agree well with those of Wang et al. (2000). Later, Vít et al. (2007) and Wu and Wang (2007) applied this value to the evaluation of experimental data.

The main aim of the present study is to investigate the horizontal air flow over a heated cylinder in the laminar vortex shedding regime. Investigations include the analyses of flow properties such as lift coefficient and Strouhal number by varying the Reynolds number and temperature ratio.

2. Numerical solution

The computational domain is characterized by two concentric circles: the inner represents the cylinder surface with diameter d, the outer the far field with diameter d_∞. The origin of the coordinate system is in the center of the cylinder. The positive x-axis is in the downstream direction.

Typical boundary conditions are used for velocity and pressure. At the inlet uniform velocity distribution (u_∞) and constant temperature are prescribed (T_∞). The cylinder surface is kept at different constant temperature (T_w), the temperature ratio range is $1 \le T^* \le 2$. Time-dependent, incompressible fluid flow and forced convection are assumed.

The accuracy of the computed flow quantities depends on the resolution (number and distribution of grid points), time step and the size and shape of the computational domains. In Bolló (2010) the solution of the flow around a circular cylinder were tested on different grids and different time steps and the optimum grid and time step were determined. The influence of computation domain size and its validation are described in detail in Bolló and Baranyi (2010). The computational grid is 360×310 (azimuthal$\times$radial) for $d_\infty / d = 220$. In the physical domain logarithmically spaced radial cells are used, providing a fine grid scale near the cylinder wall and a coarse grid in the far field. The minimal dimensionless mesh size in radial direction is 0.00875. The time step of $\Delta t = 0.001$ is used in all computations.

The commercial software FLUENT is used for the numerical simulations, based on the finite volume method. The two-dimensional, unsteady, laminar, segregated solver is used to solve the incompressible flow on the collocated grid arrangement. The Second Order Upwind Scheme was used to discretize the convective terms in the momentum equations. The semi-implicit method for the pressure linked equations (SIMPLE) scheme is applied for solving the pressure-velocity coupling.

3. Results

The finite-volume method was validated by comparing data available in literature for dimensionless coefficients. The accuracy of numerical results is compared by means of integral quantities such as lift coefficient and the Strouhal and Nusselt number. The lift coefficient is calculated as

$$C_L = \frac{2F_L}{\rho u_\infty^2 d}, \tag{2}$$

where $\rho-$ is the fluid density, [kg/m^3] F_L- is the lift, [N]. The Strouhal number, the nondimensional shedding frequency, is defined as

$$\mathrm{St} = f\,d\,/\,u_\infty, \tag{3}$$

where f – is the frequency of vortex shedding of C_L, [1/s].

The heat transfer between the cylinder and surrounding fluid is calculated using the dimensionless heat transfer coefficient, or Nusselt number

$$\mathrm{Nu} = h\,d\,/\,k \tag{4}$$

where k– is the thermal conductivity of the fluid, [W/(m K)] and $h-$ is the heat transfer coefficient, [W/(m^{2}K)]. The time average and root-mean-square (*rms*) values are defined as

$$C_{mean} = \frac{1}{nT}\int_t^{t+nT} C(t)dt\,,\; C_{rms} = \sqrt{\frac{1}{nT}\int_t^{t+nT}\left[C(t)-C_{mean}\right]^2 dt}\,, \tag{5}$$

where T – is the period of vortex shedding, [s], $n-$ is the number of periods. Here C stands for either lift coefficient or Nusselt number.

3.1 Vortex shedding frequency

At $\mathrm{Re} = 160$ the vortex shedding frequency was computed for various values of temperature ratios in the range $1 \le T^* \le 2$ shown in Fig. 1(a). As seen in the figure, the shedding frequency depends on the temperature, it decreases with increasing temperature. The Strouhal number at different temperature ratios is displayed in Fig. 1(b) together with data from the literature. For isothermal flow Williamson and Brown suggested an St-Re relation in a polynomial form: $St = 0.2665 - 1.018/\sqrt{Re}$. It can be seen that the present results agree well with experimental data for the isothermal case.

For nonisothermal flow the present results agree well with the experimental data of Wang et al. (2000). Shi et al. (2004) used the FASTEST2D numerical solution and resolved the governing equations with the finite volume method. For $T^* = 1.5$ the present work fits accurately to the curve gained from their results. The present data agree well also with the numerical results of Sabanca and Durst (2003), who used the finite volume method and took into account the compressibility effect.

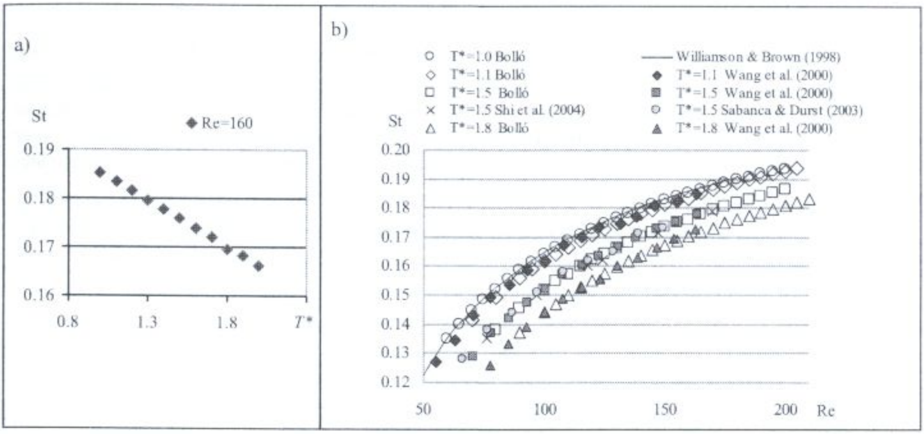

Fig. 1: St number (a) versus temperature ratio at $\mathrm{Re} = 160$ *and (b) versus Reynolds number*

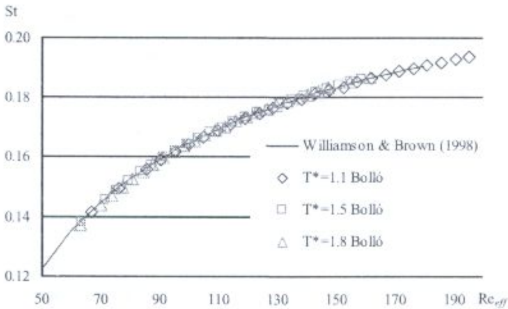

Fig. 2: The vortex shedding frequency versus effective Reynolds number

Lecordier et al. (1991) found that the onset of vortex shedding at various temperature ratios can be well characterized by an effective Reynolds number $\mathrm{Re}_{eff} = d\,u_\infty / \nu(T_{eff})$. This quantity is based on the kinematic viscosity evaluated at the effective temperature T_{eff} (Eq. (1)). Wang et al. (2000) successfully correlated their experimental data for the Strouhal number against Re_{eff} using a slightly different effective temperature, $c = 0.28$; the present results with this correlation applied are shown in Fig. 2. The data for St at $T^* = 1.1;1.5;1.8$ agree well with the curve by Williamson and Brown (1998). Hence the accuracy of the correlation of Wang et al. (2000) is confirmed.

3.2 Lift coefficient

The *rms* value of lift-coefficient fluctuations (C_{Lrms}) with temperature ratio at $\mathrm{Re}=160$ are shown in Fig. 3(a). As seen in the figure, the lift coefficient decreases with increasing temperature. In Fig. 3(b) it can be seen that the *rms* lift coefficient increases with increasing Re. The curve fit based on the isothermal data of Norberg (2001) is also included for comparison. For unheated cylinder Norberg (2001) suggests an approximate formula for the *rms* of lift coefficient ($C_{Lrms}=\sqrt{\varepsilon/30+\varepsilon^2/90}$, $\varepsilon=(Re-47)/47$). His data and those of the present work are nearly identical ($T^*=1.0$), aside from a small difference for $Re>180$. For a heated cylinder it can easily be recognized that there are systematic differences between the curves belonging to different temperature ratio values.

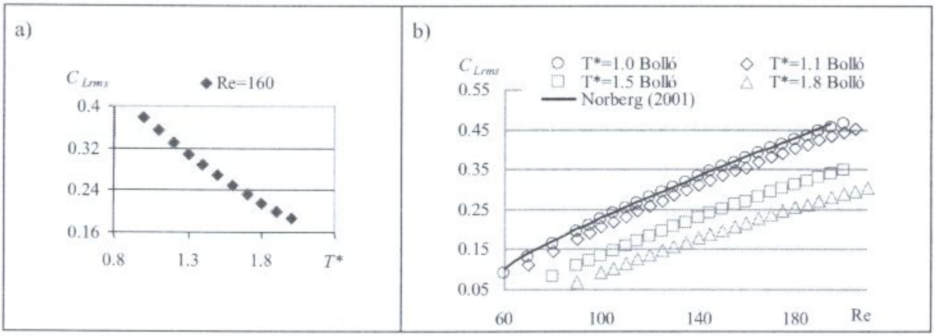

Fig. 3: C_{Lrms} (a) versus temperature ratio at $\mathrm{Re}=160$ and (b) versus Reynolds number

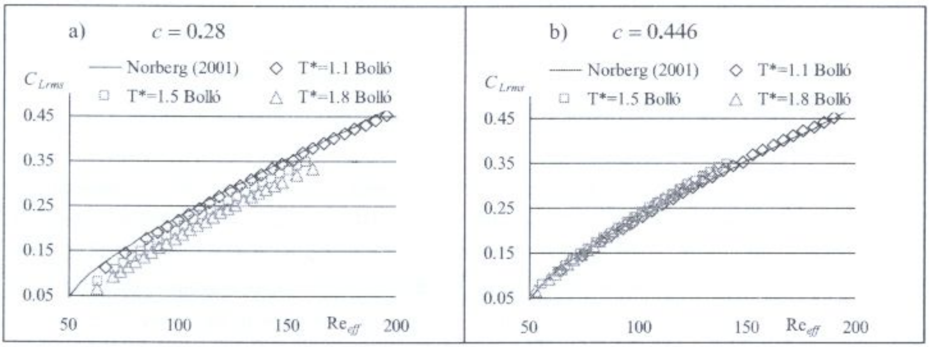

Fig. 4: C_{Lrms} versus effective Reynolds numbers with different c values

In Fig. 4(a) the lift coefficient is shown against the effective Reynolds number using $c = 0.28$. It can be seen that the present C_{Lrms} data at $c = 0.28$ do not agree with the curve by Norberg (2001). The $c = 0.28$ value proposed by Wang et al. (2000) cannot be used here. In Fig. 4 (b) C_{Lrms} is shown for $c = 0.446$. In this way the present work agrees well with the data of Norberg (2001).

3.3 Nusselt number

In this subsection the Nusselt number (Nu_f), based on film temperature ($c = 0.5$), is investigated. The distribution of local Nusselt number (Nu_f) over the cylinder surface shown at Re=150 in Fig. 5(a) provides a clear picture of the role of the temperature effect. In the figure the angle $\theta = 0°$ is defined at the front stagnation point. The local Nusselt number first decreases until it reaches a minimum value of about $\mathrm{Nu}_f = 2$ near the separation point and then slightly increases. The local Nusselt number decreases with increasing T^*, as seen in Fig. 5(a). Similar results were obtained by Baranyi (2003), who investigated force convection from the cylinder by means of the finite difference method and applied the temperature ratio ($T^* \approx 1.0$) and slightly higher Re number (Re=160). At the separation point (around $\theta \cong 140°$) the Nu values are almost identical for the three cases as well as for the data of Baranyi (2003) shown. The local Nusselt number for different Re numbers at the temperature ratio of $T^* = 1.5$ is shown in Fig. 5(b). As seen in the figure, the Nusselt number increases with increasing Re.

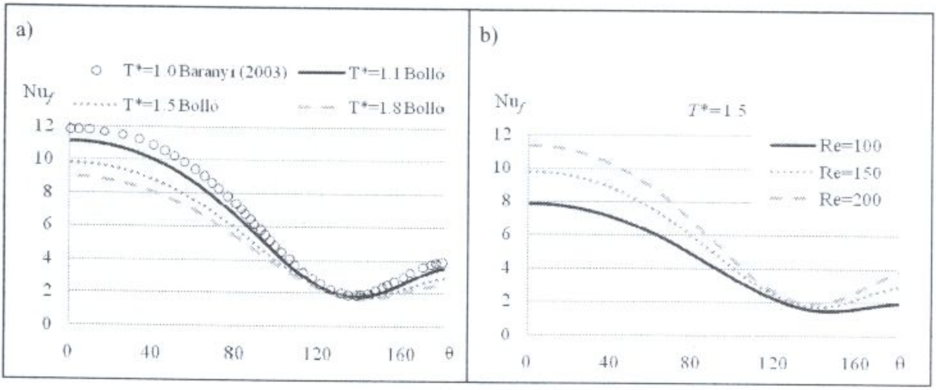

Fig. 5: The local distribution of Nu_f *(a) at* $\mathrm{Re} = 150$, *(b) for constant temperature ratio of 1.5 for three Re values*

Fig. 6(a) shows the film Nusselt number (Nu_f) versus Reynolds number for three temperature ratios. In general, the agreement of present data with the experimental data of Wang and Travnicek (2001) is good. They suggest a relationship between Nu_f and the expression of $\mathrm{Re}_f T^{*0.25}$:

$$\mathrm{Nu}_f = -0.153 + 0.527(\mathrm{Re}_f\, T^{*0.25})^{0.5}, \quad (6)$$

shown as a solid line in Fig. 6(b). The present numerical data collapse well on this curve, as can be seen in the figure. The maximum deviation between the present data and values obtained by Eq. (6) is less than 1%.

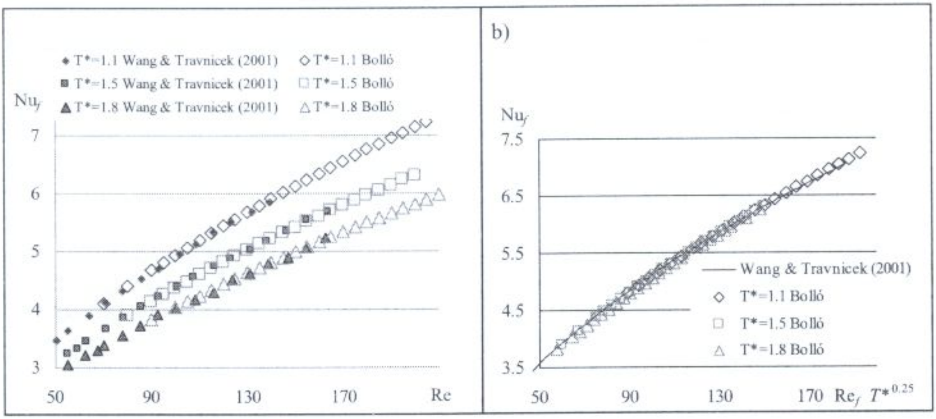

Fig. 6: (a) The film Nusselt number Nu_f *versus* Re, *(b)* Nu_f *versus* $\mathrm{Re}_f\, T^{*0.25}$

4. Conclusions

The influence of the temperature ratio on the two dimensional laminar flow of air past a heated circular cylinder was investigated based on the finite volume method. The wake behind the cylinder is sensitive to temperature ratio. The effect of heating was computed and compared with the available experimental and computational results in the literature and good agreement was found. The frequency of the vortex shedding decreases with increasing heat input. The effective temperature concept was used for describing vortex shedding from a heated cylinder. For Strouhal number the present results show that the effective temperature could be used to describe the heat transfer correlation as well. In Eq. (1) the coefficient in air was found to be $c = 0.28$, as was also found in Wang et al. (2000), Sabanca and Durst (2003) and Shi et al. (2004).

C_{Lrms} decreases with increasing temperature ratio. It was found that if we use $c = 0.446$ in Eq.(1) then C_{Lrms} values for different temperature ratios collapse on the curve for an unheated cylinder.

The local Nusselt number on the cylinder surface decreases with increasing temperature ratio at a given Reynolds number, but at the separation point Nu_f is the same for all temperature ratios. For constant temperature ratio the Nusselt number increases with increasing Re number.

Acknowledgements

The support provided by the Hungarian Research Foundation (OTKA project K 76085 and NKTH-OTKA 68207) and the Hungarian-German Intergovernmental S&T co-operation program (P-MÖB/836) is gratefully acknowledged.

References

Baranyi, L. (2003). Computation of unsteady momentum and heat transfer from a fixed circular cylinder in laminar flow, Journal of Computational and Applied Mechanics, Vol. 4, No. 1, p. 13-25.

Bolló, B. (2010). Grid independence study for flow around a stationary circular cylinder, microCAD 2010, International Computer Science Conference, Miskolc, Hungary, Section F, p. 1-6.

Bolló, B., Baranyi, L. (2010). Computation of low-Reynolds number flow around a stationary circular cylinder, 7th International Conference on Mechanical Engineering, Budapest, Hungary, p. 891-896.

Dumouchel, F., Lecordier, J.C., Paranthoën, P. (1998). The effective Reynolds number of a heated cylinder, Int. Journal of Heat Mass Transfer, Vol. 41, No. 12, p. 1787-1794.

Lecordier, J.C., Hamma, L., Paranthoën, P. (1991) The control of vortex shedding behind heated circular cylinder at low Reynolds numbers, Experiments in Fluids, Vol. 10, p. 224-229.

Norberg C. (2001). Flow around a circular cylinder: aspects of fluctuating lift, Journal of Fluids and Structures, Vol. 15, p. 459-469.

Sabanca, M., Durst, F. (2003). Flows past a tiny circular cylinder at high temperature ratios and slight compressible effects on the vortex shedding, Physics of Fluids, Vol. 15, No. 7, p. 1821-1829.

Shi, J.M., Gerlach, D., Beuer, M., Biswas, G., Durst, F. (2004). Heating effect on steady and unsteady horizontal laminar flow of air past a circular cylinder, Physics of Fluids, Vol. 16 , No. 12, p. 4331-4345.

Vít, T., Ren, M., Trávnníček, Z., Maršík, F., Rindt, C.C.M. (2007). The influence of temperature gradient on the Strouhal–Reynolds number relationship for water and air, Experimental Thermal and Fluid Science, Vol. 31, p. 751-760.

Wang, A.B., Trávníček, Z., Chia, K.C. (2000). On the relationship of effective Reynolds number and Strouhal number for the laminar vortex shedding of a heated circular cylinder, Phys. Fluids, Vol. 12 , No. 6, p. 1401-1410.

Williamson C. H. K., Brown G. L. (1998). A series in $1/\sqrt{Re}$ to represent the Strouhal-Reynolds number relationship of the cylinder wake, Journal of Fluids and Structures; Vol. 12, No. 8, p. 1073-1085.

Wu, M.H., Wang, A.B. (2007). On the transitional wake behind a heated circular cylinder. Physics of Fluids, Vol. 19, 084102.

Zdravkovich, M.M. (1997). Flow around Circular Cylinders, Vol. 1: Fundamentals. Oxford University Press, Oxford/New York.