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Describing the Pressure Dependence of Lamé Coefficients on Coal Samples

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SUMMARY

Understanding the relationship between pressure and rock physical parameters, such as acoustic velocity, elastic moduli, porosity is essential for exploring and exploiting of natural reserves. Therefore a petrophysical model for describing the pressure dependence of P and S wave velocities was developed. The advance of the model is that it provides the required physical explanation of the pressure dependence with its three-parameter exponential function. On the basis of the model the pressure dependent Lamé coefficients were deduced. To prove the applicability of our method laboratory data of coal specimen was inverted to estimate the model parameters. Since the model valid for P and S wave velocity has a common parameter a joint inversion technique was applied. The velocity data were measured under varying confining pressure with pulse transmission technique on dry samples. A very good fitting between measured and calculated data was found thus inversion results strengthened the applicability of the petrophysical model.
Introduction

Several geophysical methods are available to determine rock physical parameters, such as acoustic velocity, elastic moduli, porosity, etc. It is well known that pressure has a significant influence on these and other textural, mechanical and transport parameters. The understanding of the relationship between them is essential for exploring and exploiting of natural reserves. In connection with the acoustic velocities numerous qualitative models are available in the literature. The most common explanation of the phenomena of pressure dependence is the closure of microcracks (Walsh and Brace 1964) or the closure of pores (Birch 1960).

Equations published in literature (e.g. Singh et al. 2006, Ji et al. 2007) do not give the physical explanation of the process, they contain only the regression parameters of the best fitting functions. Contrarily in the paper the authors present a rock physical model for describing the pressure dependence of acoustic velocities, which provides the required physical connection hence the pressure dependence of Lamé coefficients can be also derived.

Effect of pressure on the acoustic velocities

The quantitative model for describing the pressure dependence of acoustic velocities presented in this paper is based on Birch’s (1960) qualitative assumption, that the main reason of the pressure dependence of acoustic wave velocities is the closure of pores. Therefore we introduce the parameter $V$ as the unit pore volume of a rock. We assume that a $d\sigma$ stress increase applied to the rock will generate a $dV$ change in pore volume directly proportional to the change in stress. The proportionality factor $-\lambda_V$ is a material dependent rock physical parameter. One can describe these assumptions with the following differential equation and its solution

$$dV = -\lambda_V V d\sigma \implies V = V_0 \exp(-\lambda_V \sigma),$$

(1)

where $V_0$ is the pore volume at stress-free state ($\sigma = 0$). The negative sign represents that the increasing stress decrease the pore volume. The Eq. (1) describes inner rock physical relation. It means that it does not depend on the type of the acoustic wave which propagates through the rock. It can be applied also for the P- and S-waves. We assume also a linear relationship between the infinitesimal change of the propagation wave velocity ($d\alpha$ for P-wave and $d\beta$ for S-wave) and $dV$

$$d\alpha = -\kappa_P dV, \quad d\beta = -\kappa_S dV,$$

(2)

where $\kappa_P$ and $\kappa_S$ are proportionality factors, new material characteristics respectively for P and S waves. The negative sign represents that the velocity is increasing with decreasing pore volume. Combining the assumptions of Eqs. (1-2) and solve the differential equation one can obtain

$$d\alpha = \kappa_P \lambda_V V_0 \exp(-\lambda_V \sigma) d\sigma \implies \alpha = K_1 - \kappa_P V_0 \exp(-\lambda_V \sigma),$$

$$d\beta = \kappa_S \lambda_V V_0 \exp(-\lambda_V \sigma) d\sigma \implies \beta = K_2 - \kappa_S V_0 \exp(-\lambda_V \sigma),$$

(3)

where $K_1$ and $K_2$ are integration constants. At stress-free state ($\sigma=0$) the propagation velocities ($\alpha_0$ and $\beta_0$) can be measured and $K_1$, $K_2$ can be computed from Eq. (3) as $\alpha_0 = K_1 - \kappa_P V_0$ and $\beta_0 = K_2 - \kappa_S V_0$. With introducing the notations $\Delta\alpha_0 = \kappa_P V_0$, $\Delta\beta_0 = \kappa_S V_0$ Eq. (3) can be rewritten in the following forms

$$\alpha = \alpha_0 + \Delta\alpha_0 (1 - \exp(-\lambda_V \sigma)), \quad \beta = \beta_0 + \Delta\beta_0 (1 - \exp(-\lambda_V \sigma)).$$

(4)

Eq. (4) provides a theoretical connection between the propagation velocity and rock pressure. In the framework of the model, the velocity of acoustic wave increases from $\alpha_0$ (at zero pressure) to $\alpha_{\text{max}} = \alpha_0 + \Delta\alpha_0$ (at high pressure, when all the pores are closed) and from $\beta_0$ to $\beta_{\text{max}} = \beta_0 + \Delta\beta_0$. So, $\Delta\alpha_0$ and $\Delta\beta_0$ can be considered the velocity-drops caused by the presence of pores at zero pressure (Ji et al. 2007). Note that in the range of high pressures, reaching a critical pressure the reversible range is exceeded.
and destruction of the sample may occur thus decreasing velocity can be observed. This effect is outside of our present investigations. Therefore this model is valid only in the reversible range. As it can be seen from the Eq. (4) the \( \lambda V \) is a common petrophysical parameter. To prove this, the P- and S-wave velocity data were determined in two independent inversion procedures and the results established this statement. Therefore it was possible to process the data by joint inversion technique. The physical meaning of \( \lambda V \) can be given by introducing the notation \( \Delta \alpha = \alpha_{\text{max}} - \alpha \), (the velocity-drop caused by the presence of pores at pressure \( \sigma \)) Eq. (4) can be written in the form

\[
\Delta \alpha = \Delta \alpha_0 \exp(-\lambda V \sigma). \tag{5}
\]

Laboratory tests indicate that the variant types of rock response in a different scale to pressure change. This feature can be described with the sensitivity function, which is widely used in literature. Hence we introduce the (logarithmic) stress sensitivity of the velocity-drop as

\[
S(\sigma) = \frac{1}{\Delta \alpha} \frac{d \Delta \alpha}{d \sigma} = \frac{d \ln(\Delta \alpha)}{d \sigma} \rightarrow \lambda V = -\frac{d \ln(\Delta \alpha)}{d \sigma} = S. \tag{6}
\]

By using Eq. (5) it can be seen that the petrophysical characteristic \( \lambda V \) is the logarithmic stress sensitivity of the velocity-drop (Dobróka and Somogyi Molnár 2012).

**Determination of pressure-dependent Lamé coefficients**

In the frame of the perfectly elastic medium model (in which the Hooke’s law is a special border-line case) the Lamé constants are considered as quantities locally interpretable in the given deformation or rather stress state. After determining the longitudinal (\( \alpha \)) and transverse (\( \beta \)) wave velocities as well as the density (\( \rho \)) of the medium, the Lamé coefficients can be calculated with the formulas

\[
\mu = \beta^2 \rho, \quad \lambda = \alpha^2 \rho - 2\mu, \tag{7}
\]

where \( \mu \) is the first and \( \lambda \) is the second Lamé coefficient. The influence of pressure on density is negligible in comparison to the effects on velocity. It means the changes in velocities can be transmitted to the changes in the Lamé coefficients. Therefore after estimation of the model parameters and calculation of velocities, the pressure dependence of Lamé coefficients can be also deduced. Based on the \( \mu \) and the \( \lambda \), other moduli used in rock mechanic can be derived, such as bulk modulus, shear modulus, Young modulus, Poisson number.

**Samples**

The applicability of the method was tested on data sets published in literature. Yu et al. (1993) measured P- and S-wave velocity data on coal samples using pulse transmission technique. The confining pressure was increased from 2 MPa to 40 MPa, the pore pressure was held constant and equal to atmospheric pressure. The specimen No. 22 and 32 were chosen for this study. Their descriptions are summarized in Table 1.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Length (mm)</th>
<th>Diameter (mm)</th>
<th>( \rho_{\text{dry}} ) (kg/m(^3))</th>
<th>Porosity (%)</th>
<th>Petrological description</th>
<th>Orientation to bedding</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. 22</td>
<td>49,8</td>
<td>60,7</td>
<td>1,36</td>
<td>1,7</td>
<td>dull plus bright bands</td>
<td>perpendicular</td>
</tr>
<tr>
<td>No. 32</td>
<td>49,7</td>
<td>60,7</td>
<td>1,35</td>
<td>1,5</td>
<td>parallel</td>
<td></td>
</tr>
</tbody>
</table>

**Case studies**

The presented rock physical model contains five model parameters (\( \alpha_0, \Delta \alpha_0, \beta_0, \Delta \beta_0, \lambda V \)) and as it was mentioned \( \lambda V \) is a common petrophysical parameter. Their values were determined from measured data by means of joint inversion method. Since the number of measured data was much more than the
parameters to be estimated the principle of least squares method was used. The results are shown in Table 2. With the presented estimated model parameters one can calculate the velocities at any pressure. Based on these values the pressure-dependent Lamé coefficients can be derived by using Eq. (7).

Table 2 Estimated model parameters by joint inversion method.

<table>
<thead>
<tr>
<th>Sample</th>
<th>P wave</th>
<th>Common parameter</th>
<th>S wave</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\alpha_0$ (m/s)</td>
<td>$\Delta \alpha_0$ (m/s)</td>
<td>$\lambda_V$ (1/MPa)</td>
</tr>
<tr>
<td>No. 22</td>
<td>2345,21</td>
<td>272,13</td>
<td>0,0921</td>
</tr>
<tr>
<td>No. 32</td>
<td>2461,48</td>
<td>345,65</td>
<td>0,1248</td>
</tr>
</tbody>
</table>

The results can be seen in Figures 1-2 in which solid lines show the calculated data. In the graph of velocities the dots represent the measured data. In case of the Lamé coefficients the “measured” dots indicate the values calculated from the measured velocities. Minimal difference between measured and calculated data can be seen in the figures.

![Figure 1](image1.png)
**Figure 1** The pressure dependence of velocities and Lamé coefficients for the sample No. 22.

![Figure 2](image2.png)
**Figure 2** The pressure dependence of velocities and Lamé coefficients for the sample No. 32.
For the characterization of the accuracy of inversion estimates the RMS ($D$) value and the mean spread ($S$) were calculated by the following formulas

$$ D = \sqrt{\frac{1}{N} \sum_{k=1}^{N} (d_k^{(m)} - d_k^{(c)})^2 \cdot 100[\%]}, \quad S = \sqrt{\frac{1}{M(M-1)} \sum_{i=1}^{M} \sum_{j=1}^{M} (\text{corr(m)}_{ij} - \delta_{ij})^2}. \quad (8) $$

Table 3 contains these values for each sample in the last iteration step. It can be seen that the data misfits (RMS) were small and the mean spread values indicate that the parameters are in moderate correlation, so the inversion results are reliable. These results confirm the accuracy of the inversion estimates and the feasibility of the developed petrophysical model in practice.

<table>
<thead>
<tr>
<th>Sample</th>
<th>$D_{\text{velocity}}$ (%)</th>
<th>$D_\mu$ (%)</th>
<th>$D_\lambda$ (%)</th>
<th>$S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. 22</td>
<td>0,1986</td>
<td>0,4188</td>
<td>0,4768</td>
<td>0,5459</td>
</tr>
<tr>
<td>No. 32</td>
<td>0,4312</td>
<td>1,0594</td>
<td>1,0340</td>
<td>0,5936</td>
</tr>
</tbody>
</table>

Conclusions

The authors presented a new model for describing the pressure dependence of acoustic velocities and Lamé coefficients. The data can be processed by using joint inversion method, where two equations should be solved during forward modelling: $\alpha = \alpha_0 + \Delta\alpha_0 \cdot (1-\exp(-\lambda \sigma))$ for the longitudinal wave velocity and $\beta = \beta_0 + \Delta\beta_0 \cdot (1-\exp(-\lambda \sigma))$ for the transverse wave velocity. Since $\lambda$ is a common material dependent parameter, five model parameters ($\alpha_0, \Delta\alpha_0, \beta_0, \Delta\beta_0, \lambda$) are estimated during the calculations. After determination of velocities, the pressure dependence of Lamé coefficients can be deduced. The advance of the model is that it delivers the required physical explanation of the pressure dependence. To prove the applicability we tested our model on laboratory data of coal specimen chosen from literature. Very good accordance between measured and calculated data was found. Inversion results confirmed the accuracy and feasibility of the petrophysical model.

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References


