Static and dynamic analyses of composite beams with interlayer slip

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Synopsis of PhD theses

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1 Preliminaries

Layered composite structures, especially layered beams are widely applied in building and bridge engineering since the advantages of the layers made of different elastic materials can be well married, while their disadvantages can be reduced or eliminated. Therefore it is very important to understand the mechanical behaviour of the layered composite beams and the influence of the connection between the layers for the mechanical properties. In some cases it is assumed that the connection is perfect both in normal and tangential direction and this assumption provides satisfying results for these problems. The theory of this kind of composite beams is well developed. However, in a lot of other cases it is necessary to deviate from this assumption. Namely the beam components are generally joined to each other by different shear connectors such as nails, studs, screws or rivets. Because of the elastic deformation of these connectors two phenomena can occur among the layers. In normal direction the beam components may be divorce and in tangential direction an interlayer slip can happen. The experiments and measurements have proven that the effects of these phenomena cannot be neglected in a number of cases. This thesis is restricted to that problems when the connection is perfect in normal direction (the divorce of the layers is not allowed) but there is interlayer slip in tangential direction.

One of the most commonly used configuration is shown in Fig. 1. This type of composite beam is widely applied in the bridge industry. Its cross-section consists of a concrete slab with steel reinforcement and a steel joist. A large amount of studies and researches deal with this configuration and its mechanical behaviour. Composite structures are also utilized as floor and wall elements, e.g. timber-concrete elements composed of thin concrete plates attached to wood studs by means of shear connectors. The common property of these structural elements is the interlayer slip.

The first analytical works analysing the behaviour of composite beams with weak shear connection appeared in the 40’s and 50’s [1–3]. The pioneering and most cited work is definitely paper by Newmark
et al. [1]. Their model, which is called the Newmark’s model in the literature, used the following assumptions (i) the layers have linear elastic materials, (ii) the layers separately follow the Euler-Bernoulli beam theory, (iii) the vertical separation of the layers is not allowed. The problem was governed by a linear differential equation of second order in the longitudinal force resisted by the top element, and the other unknowns were the longitudinal force and the expression for moment along the beam.

In the late 60’s, Goodman and Popov [4] further developed the Newmark’s model and extended it for three-layered wood beams with interlayer slip. They deduced a differential equation of fourth order in the deflection and also contained the expression of the moment along the beam (the latter one is only unknown in the case of indeterminate beam). They consider the problem with one concentrated force applied at mid span and two concentrated force at third points. Adekola [5] proposed a model which took into account the vertical separation (it is also called uplift) of the layers and frictional effects. For the computations Adekola applied the finite difference method to solve the problem numerically. Other researchers further investigated the influence of the uplift [6–8]. They computed the error caused by neglecting the uplift in the Newmark’s model and determined that the effect of the uplift can be ignored since the order of this error is a few percent.

Girhammar and Gopu [9] proposed a formulation for the exact
first- and second-order analyses of composite beam-columns with partial shear interaction subjected to transverse and axial loading. In this study the authors extended the Newmark’s model with taking into account axial loading. The governing differential equation was of the sixth order in vertical displacement. Ecsedi and Baksa [10] also deduced the governing equation of the problem in terms of the slip and the vertical displacement. Previous researches including the work of Newmark et al. [1] always assumed no axial force on the composite beam. Girhammar and Pan [11] developed a model for the exact and approximate analysis for composite beams with interlayer slip subjected to general dynamic load.

Some study dealt with the behaviour of continuous composite beams with interlayer slip in the linear-elastic range [12–14]. Plum and Horne [12] investigated a two-span continuous beam subjected to two equal point loads at the centres of the two span. They proposed closed-form solutions for the deflection, for the longitudinal force in the top element, for the slip, for the slip strain and for the redundant moment at the internal support. A two-span and a three-span continuous beam were analysed by Jasim [13]. The two-span continuous beam was subjected to both distributed line load and point loads at mid-spans, whilst the three-span continuous beam had a point load at the centre of the internal span. Jasim and Atalla [14] provided a simplified solution to determine the deflection of a continuous composite beam. However, the formulation can be derived for the continuous beams based on the Newmark’s model, the computations can easily become lengthy and difficult.

Faella et al. [15, 16] developed a stiffness element with 6 dof which are the vertical displacement, the rotation and the slip at both ends. To obtain the stiffness matrix the flexibility matrix was inverted for the case of a simply supported beam. The flexibility coefficients had already been derived by Consenza and Pecce [17]. The determination of the stiffness matrix based on the Newmark’s model, thus this method is able to provide the same results as the solution of the governing equation of the partial shear interaction problem. All the above mentioned study dealt with the problem of composite beams with interlayer slip in linear-elastic range. In this thesis the problem of the considered composite
beams is also analysed in the linear-elastic range.

An excellent thesis was carried out by Ranzi [18] on composite beams with partial shear interaction. The author utilized and further developed several models mentioned above. Namely general solution was derived for two- and for \( m \)-layered composite beams with inter-layer slip in linear elastic range. Some stiffness elements were also described to analyse the problem of steel-concrete composite beams. The author summarized several theories of material non-linearity which were also applied. Finally time analyses were introduced including the time-dependent behaviour of concrete assuming both full and partial shear interaction between the layers. The steel-concrete composite beam analysed by Ranzi had the cross-section shown in Fig. 1. The author also provided a detailed and useful literature review.

Other beam theories were adopted for investigation of composite beams with interlayer slip as well. The Timoshenko beam theory was used in [19]. Murakami [19] formulated boundary value problems by means of the principle of virtual work. Combining the development of finite elements with the Timoshenko beam theory was also analysed in [20–25]. Recently several works have also revealed utilizing higher order beam theories for the problem of composite beams with partial shear interaction [26–29].

There exist several works in connection with the dynamic analysis of composite beams with interlayer slip [11, 30–33]. An exact and an approximate analysis of composite members with partial interaction and subjected to general dynamic loading were presented by Girhammar and Pan [11]. Adam et al. [30] analysed the flexural vibration of composite beams with interlayer slip using the Euler-Bernoulli beam theory. The governing sixth-order initial-boundary value problem was solved by separating the dynamic response in a quasi-static and in a complementary dynamic response. Heuer and Adam extended the previous model for composite beams made of piezoelectric materials in [31]. The partial differential equations and general solutions for the deflection and internal actions and the pertaining consistent boundary conditions were presented for composite Euler–Bernoulli members with interlayer slip subjected to general dynamic loading in [32]. Wu et al. [33] derived
the governing differential equations of motion for the partial-interaction composite members with axial force. All these works neglected the influence of the axial and rotary inertia.

The elastic stability problems of composite beams with weak shear connection were also investigated [34–38]. Challamel and Girhammar [34] analysed the lateral-torsional stability of vertically layered composite beams with interlayer slip based on a variational approach. An analytical method was presented for the delamination buckling using the Timoshenko beam theory by Chen and Qiao [35]. Grogne et al. [36] utilized the Timoshenko beam theory as well. Schnabl and Planinc [37] presented a detailed analysis of the influence of boundary conditions and axial deformation on the critical buckling loads and the same authors took into account the effect of the transverse shear deformation on the buckling [38].

Although a lot of papers were published in connection with layered curved beams with perfect shear connection, only a few works counted the influence of the interlayer slip [39–41]. For out-of-plane deformation and loads the time dependent creep and shrinkage behaviour of horizontally curved steel-concrete composite beams with partial shear interaction were analysed by Liu et al. [39]. Erkmen et al. [40] developed a total Lagrangian finite element formulation for elastic analysis of steel-concrete curved composite beams. A three-dimensional finite element model is used to simulate composite steel-concrete curved beams subjected to combined flexure and torsion [41]. Tan and Uy gave a detailed description of the torsion induced vertical slip [41].

2 Objectives

According to the literature review one can see that a number of investigators dealt with the static analysis of composite beams with interlayer slip to determine the governing equation of the problem. In many cases the analysis led to a higher order differential equation the solution of which is often difficult and cumbersome. Thus it is my

Objective 1 to provide an analytical method for the solution of the governing equation the application of which is handy and needs less
computations. In connection with this objective I draw up the following items:

• to write the governing equation of the problem in terms of the slip and shear force function,

• to deduce the so-called fundamental solutions for the problem by means of both the Euler-Bernoulli and the Timoshenko beam theory,

• to apply the developed solutions for various beams and boundary conditions and compare with results derived from other studies and from FEM solution.

The overview of the literature shows the lack of researches in accordance with composite beams with interlayer slip under the action of thermal loading. My

Objective 2 is to take into account the effect of thermal loading with the help of the following items:

• to derive the governing equation of the problem in terms of the slip and shear force function using the Euler-Bernoulli beam theory and the Duhamel-Neumann’s law,

• to determine the solution of the governing equation with various boundary conditions,

• to provide formulae for the computation of the stresses.

A very important question is the stability analysis of composite beams with weak shear connection.

Objective 3 is to analyse the buckling of the composite beams, namely

• I aim to determine the buckling load based on the principle of minimum potential energy,

• I also intend to deduce the buckling load by dint of exact analysis to compare with the variational method,
• My further purpose is to give the function of the buckling load in terms of the slip modulus.

However, there are many studies on vibration analysis of composite beams in relation to the free flexural vibration, these works neglect the effect of the rotary and axial inertia. My

Objective 4 consists of the following items:

• to deduce the equations of motion including the d’Alembert forces taking into account the rotary and axial inertia

• to provide a closed form solution for the eigenfrequencies of the composite beam.

The literature contains a number of researches analysing layered curved composite beams with perfect shear connection, but there exist only a few works on the effect of the partial shear interaction. The

Objective 5 of the thesis includes

• developing an analytical method based on the principle of minimum potential energy to describe the behaviour of curved composite beams with interlayer slip,

• writing a Rayleigh-Betti type reciprocity relation for the considered curved composite beams.

3 Novel results

In this thesis I have dealt with static and dynamic problems of layered composite beams having not perfect connection. The overview of the literature represents that a lot of investigators published their works in connection with this topic in the last 60-70 years. The importance of the topic is well illustrated by the fact that the scientists research the behaviour of the composite beams with weak shear connection nowadays as well. According to the publications available in the open literature I have been able to draw up my objectives and in the following I am going to summarize the novel results of this thesis.
Statement 1.
I have derived a novel analytical solution to describe the static behaviour of composite beams with interlayer slip. The governing equation of the problem is written in terms of the slip and the cross-sectional shear force function. The fundamental solutions for seven different initial conditions have been deduced by means of both Euler-Bernoulli and Timoshenko beam theory. With these functions the solution of the governing equation have become the solution of a linear system of equation. I have presented the methods in numerical examples with different boundary conditions in order to compare the results with ones from other publications and from my FEM analysis. The results were in good agreement.

Statement 2.
I have deduced a new analytical solution for static problem of composite beams with interlayer slip loaded by mechanical and thermal load as uniform temperature change. I have provided the governing equation of the problem and have solved it for different boundary conditions. The thermal stresses have been derived as well. In this case numerical examples also represented the developed method with and without thermal loading. The same results have been obtained from this method without thermal load as from the fundamental solutions.

Statement 3.
Two new analytical method have been formulated for the determination of the buckling load of composite beams with weak shear connection. In the first case closed form solution were derived from a variational method for two composite columns with different boundary conditions. In the second case I have obtained the same closed forms from the equilibrium method for the same two composite columns. Two numerical examples showed the application of the forms which were in good agreement with the results come from the literature. I have also provided the function of the buckling load in terms of the slip modulus for
Statement 4.

A new analytical solution has been described in connection with the free flexural vibration of composite beams with weak shear connection. For the analysis I have introduced the d’Alembert forces. According to the obtained equations of motion three closed form solutions were provided for the eigenfrequencies of the considered composite beam. The first solution counts with the effect of the rotary and axial inertia resulting three various eigenfrequencies, whilst the second one neglected the rotary inertia, the third one eliminated all the rotary and the axial inertia. The latter two resulted one eigenfrequency.

Statement 5.

A new analytical method has been elaborated for static analysis of uniformly curved composite beams with interlayer slip. Based on the Rayleigh-Betti type reciprocity relation I have deduced the potential energy of the considered uniformly curved beam. By means of the principle of minimum potential energy I have also determined the equilibrium equations and the dynamic boundary conditions. Several numerical examples represented the applications of the potential energy combining with the Ritz method and for the checking of this method the Rayleigh-Betti type reciprocity relation were used. These results were in good agreement.

4 Possible applications and future research

The developed analytical methods can be well utilized in the building and bridge industry during the design process. The results derived from the elaborated methods can be applied as benchmark solution to validate solutions from other numerical computations such as finite element method or finite differences method.

In the future a possible and interesting topic would be the dynamic analysis of composite beams subjected to axial load. It is an important
question to determine the effect of the axial load upon the eigenfrequencies.

Another research direction could be the thermal loading. It would be interesting to deal with the influence of nonuniform temperature change on the behaviour of composite beams with weak shear connection.

According to the open literature there are few publications in connection with curved composite beams with interlayer slip. Further investigation of curved composite beams including frequency analysis, stability analysis etc. might be worthy to deal with.

Future research field would also be the static and dynamic analysis of composite plates and shells with interlayer slip.

5 The candidate’s relevant publications

Articles in journals


Conference papers


(12) Á. J. Lengyel and I. Ecsedi. Kompozit rudak vizsgálata energia módszer segítségével (Analysis of composite beams by means of
energy method, in Hungarian), XII. Magyar Mechanikai Konferencia (XII. Hungarian Conference of Mechanics), Section: 12. Tartószerkezetek II., University of Miskolc, 25.08.2015–27.08.2015, paper 279.


(17) Á. J. Lengyel and I. Ecsedi. Vibrations of Composite Beams with Interlayer Slip, 8th International Conference of PhD Students, Section: D – Engineering Science, University of Miskolc, 05.08.2012–11.08.2012, paper D-10

(18) Á. J. Lengyel and I. Ecsedi. Kompozit rudak lineáris analízise Ritz-módszer segítségével (Linear analysis of composite
beams with weak shear connection by means of Ritz method),


References


[23] Q. Nguyen, E. Martinelli, and M. Hjiaj. Derivation of the exact stiffness matrix for two-layer Timoshenko beam element with par-


