IMPROVEMENT OF A SEISMIC Q MODEL

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Abstract: In this study, rock physical models are introduced to describe the pressure dependence of the seismic quality factor. The first model connects this phenomenon to the presence of one single inner process during loading of the rock samples that is manifested in the change of one extensive quantity, the pore volume. Therefore, it is called “Q-\(\sigma\)” Single Relaxation Model (SRM). Testing of the model proved that it works well in practice, i.e. the results are precise enough for engineering tasks, but in some cases it can be seen that a further development of the model could bring about a much better description of the pressure dependence of acoustic characteristics. Therefore, a new model handling the effects of two relaxation processes – the “Q-\(\sigma\)” Double Relaxation Model – is introduced as well. The model’s applicability is tested on measured (laboratory) quality factor data published in the literature. The quality checked inversion results show that the misfits between measured and calculated data become much smaller, proving that the proposed new Double Relaxation Model can be applied well in practice and provide a better understanding of pressure dependence.

Keywords: quality factor, relaxation model, single relaxation, double relaxation, pressure dependence

INTRODUCTION

The investigations of acoustic parameters such as acoustic wave velocity and quality factor started more than a decade ago at the Department of Geophysics, University of Miskolc. The aim of the research was to find new rock physical models that can describe the pressure dependence of these parameters quantitatively and obtain an explanation for the physical relation between the parameters. The basic concepts link the pressure dependence of velocities and quality factors to the change in pore volume or closure of microcracks due to the increasing load, as Birch [1] or Walsh and Brace [2] proposed in their studies. This paper focuses on investigation of the pressure dependent quality factor. When increasing pressure acts on rock, grains become closer to each other (compaction occurs in the grain structure), resulting in decreasing pore volume, which causes an increase in quality factors. This is not a sudden event, but a slow process. Therefore the model presented here is considered a relaxation model and it is valid in the framework of the constant Q model. As a first approximation, only this effect was included [3]. Since the model considered one
process – change in pore volume – as the only phenomenon influencing pressure dependent quality factor, it was called the “Q-σ” Single Relaxation Model (SRM).

“Q-σ” SINGLE RELAXATION MODEL

As outlined above, the stress increase \((d\sigma)\) causes a decrease in the value of an extensive quantity \(X\) according to the differential equation

\[
dX = -\lambda d\sigma ,
\]

(1)

where \(\lambda\) is the proportionality factor. In the Single Relaxation Model the pore volume \((dV)\) is substituted as the extensive quantity

\[
dV = -\lambda d\sigma .
\]

(2)

Let us assume a linear relationship between the change of pore volume \((dV)\) and the change of quality factors \((dQ)\) and introduce Equation (3) as model law

\[
dQ = -\kappa dV ,
\]

(3)

where \(\kappa\) is a rock physical parameter and the negative signs represent that the decreasing pore volume results in increasing quality factor. Combining the solution of Equation (2) with Equation (3) the following relations can be written

\[
dQ = \kappa V_0 e^{-\lambda \sigma} d\sigma .
\]

(4)

The quality factor at a stress-free state \((Q_0)\) can be measured, and so the integration constant can be calculated. Introducing the notation \(\Delta Q_0 = \kappa V_0\), Equation (4) receives the form

\[
Q = Q_0 + \Delta Q_0 (1 - e^{-\lambda \sigma}) ,
\]

(5)

where \(\lambda\) is the material dependent relaxation parameter. It can be seen from the three-parameter model equation that the quality factor changes exponentially with the pressure. \(\Delta Q_0\) means the quality factor drop, i.e. the difference between the quality factor at the stress-free state and at maximal stress \((Q_{\text{max}} , Q_{\beta \text{ max}})\), when all pores are closed.
“Q-σ” DOUBLE RELAXATION MODEL

The model in Section 3 describes the main characteristics of the process [3], but it was found that some samples showed a slightly different behaviour, with the measured and estimated velocities having a somewhat larger data distance in the small and high-pressure ranges. (We also have to mention that since the determination of the quality factor is much more challenging from the point of view of measurement techniques, therefore the measured data themselves contain more error.) Thus, the further development of the model seemed to be advantageous. If the rock is exposed to increasing load, several different events may arise simultaneously: closure of microcracks and spherical pore space, friction on grain or crack surfaces, compression of matrix particles, etc. These intrinsic processes can be linked to extensive quantities that show pressure dependence. Thus, a new model is introduced to handle the effects of two pressure dependent relaxation processes. It is shown in this study that the new “Q-σ” Double Relaxation Model (DRM) characterizes the pressure dependence of quality factors more accurately than the Single Relaxation Model.

The extensive quantities \(( X_1, X_2 )\) introduced in Equation (1) have effects on the quality factors as well, therefore in addition to Equation (1) the following differential equation

\[
dQ_i = -\kappa_i dX_i,
\]

is also valid, where \(i = 1, 2\) and \(\kappa_i\) is a proportionality factor, a new material characteristic. Equation (1) declares that an increase in pressure results in the decrease of extensives, where the relaxation parameter is \(\lambda\). Merging the solution of Equation (1) and Equation (6) we get

\[
dQ_1 = \kappa_1 \lambda_1 X_{01} \exp(-\lambda_1 \sigma) d\sigma
\]

\[
dQ_2 = \kappa_2 \lambda_2 X_{02} \exp(-\lambda_2 \sigma) d\sigma
\]

We are looking for the total effect of the arising intrinsic processes, i.e.

\[
dQ = dQ_1 + dQ_2 = (\kappa_1 \lambda_1 X_{01} \exp(-\lambda_1 \sigma) + \kappa_2 \lambda_2 X_{02} \exp(-\lambda_2 \sigma)) d\sigma,
\]

and its solution can be defined as

\[
Q = K - \kappa_1 X_{01} \exp(-\lambda_1 \sigma) - \kappa_2 X_{02} \exp(-\lambda_2 \sigma),
\]

where \(K\) is the integration constant, which can be calculated according to \(K = Q_0 + \kappa_1 X_{01} + \kappa_2 X_{02}\) (\(Q_0\) is the quality factor measured at stress-free state, \(\sigma = 0\)).
The notations $\Delta Q_1 = \kappa_1 X_{\sigma_0}$ and $\Delta Q_2 = \kappa_2 X_{\sigma_0}$ as quality factor increments are introduced for the two relaxation processes. With them Equation (7) can be rewritten as

$$Q = Q_0 + \Delta Q_1 (1 - \exp(-\lambda_1 \sigma)) - \Delta Q_2 (1 - \exp(-\lambda_2 \sigma)).$$

The model is valid in the reversible range and it assumes that the quality factor increases from the $Q_0$ value to the $Q_{\text{max}}$ quality factor at maximal pressure. This quantity can be written as the sum of quality factor at the stress-free state and the quality factor increments caused by the pressure increase $Q_{\text{max}} = Q_0 + \Delta Q_1 + \Delta Q_2$.

Applying this formula, the model equation can be defined

$$Q = Q_{\text{max}} - \Delta Q_1 \exp(-\lambda_1 \sigma) - \Delta Q_2 \exp(-\lambda_2 \sigma).$$

Equation (8) means the forward problem, which is solved in the framework of a quality-checked inversion procedure. The solution provides the estimates for the model parameters $Q_{\text{max}}$, $\Delta Q_1$ and $\Delta Q_2$, $\lambda_1$ and $\lambda_2$.

**APPLICATION OF THE MODELS**

To show the improvement and benefit of the new model, laboratory P-wave quality factor data were processed according to both the Single and Double Relaxation Models. In the case of the SRM 3 model parameters $(Q_0, \Delta Q_0, \lambda)$ of Equation (5) and for the DRM 5 model parameters $(Q_{\text{max}}, \Delta Q_1, \Delta Q_2, \lambda_1, \lambda_2)$ in Equation (8) were estimated in separate inversion procedures. Since the inverse problem is overdetermined (the number of data $N$ is greater than the number of model parameters $M$) the Gaussian Least Square Method was used. The model parameters used in the last iteration step are considered as the results of the inversion procedure. After determining the model parameters in the inversion procedure, the quality factors can be calculated for any arbitrary stresses.

The variance of the $i$-th estimated model parameter ($\text{var}_i$) is given by the $i$-th elements of the main diagonal in the covariance matrix ($\text{cov}(m)$)

$$\text{var}_i = \sqrt{\text{cov}(m)}_i.$$

The accuracy of the inversion can be described with the misfit between the measured and calculated data, which is given by the relative distance in data space

$$D = \frac{1}{N} \sum_{i=1}^{N} \left( \frac{d_i^{(m)} - d_i^{(c)}}{d_i^{(c)}} \right)^2 \cdot 100 \%,$$

where $d_i^{(m)}$ is the $i$-th measured data and $d_i^{(c)}$ is the $i$-th calculated data.
For the determination of the P wave quality factors, data measured by Yu et al. [4] on two coal samples were processed. The “Coal-15” sample was investigated under saturated condition, and “Coal-16” was a dry sample with density 1.37 g/cm$^3$ and porosity 2.31%. The results are plotted in Figures 1 and 2: the SRM curve with blue on the left-hand side, the DRM curve with green on the right-hand side. Red dots indicate the measured quality factors. The estimated parameters and their errors, and the relative data distance are shown in Table 1 (DRM) and Table 2 (SRM). As can be seen graphically and from the relative data distances, the new Double Relaxation Model provides a more precise description of the pressure dependence of quality factors.

Figure 1. Pressure dependent P wave quality factor of the wet sample “Coal-15” determined with the SRM (left curve) and DRM (right curve) Measured data (red dots) by Yu et al. [4]

Figure 2. Pressure dependent P wave quality factor of the dry sample “Coal-16” determined with the SRM (blue curve) and DRM (green curve) Measured data (red dots) by Yu et al. [4]
Table 1

Estimated model parameters with their error, relative data distances in the last iteration step (D) with the use of the “Q-σ” SRM

<table>
<thead>
<tr>
<th>Sample</th>
<th>$Q_0$ [-]</th>
<th>$\Delta Q_0$ [-]</th>
<th>$\lambda_1$ [1/MPa]</th>
<th>D [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coal – 15 (wet)</td>
<td>24.65</td>
<td>35.60</td>
<td>0.1122</td>
<td>1.95</td>
</tr>
<tr>
<td></td>
<td>(±0.78)</td>
<td>(±0.79)</td>
<td>(±0.0064)</td>
<td></td>
</tr>
<tr>
<td>Coal – 16 (dry)</td>
<td>7.22</td>
<td>43.01</td>
<td>0.0541</td>
<td>4.30</td>
</tr>
<tr>
<td></td>
<td>(±1.14)</td>
<td>(±0.05)</td>
<td>(±0.0060)</td>
<td></td>
</tr>
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</table>

Table 2

Estimated model parameters with their error, relative data distances in the last iteration step (D) with the use of the “Q-σ” DRM

<table>
<thead>
<tr>
<th>Sample</th>
<th>$Q_{max}$ [-]</th>
<th>$\Delta Q_1$ [-]</th>
<th>$\Delta Q_2$ [-]</th>
<th>$\lambda_1$ [1/MPa]</th>
<th>$\lambda_2$ [1/MPa]</th>
<th>D [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coal – 15 (wet)</td>
<td>69.33</td>
<td>24.88</td>
<td>20.79</td>
<td>0.1808</td>
<td>0.0232</td>
<td>1.53</td>
</tr>
<tr>
<td></td>
<td>(±20.77)</td>
<td>(±9.14)</td>
<td>(±12.02)</td>
<td>(±0.0510)</td>
<td>(±0.0480)</td>
<td></td>
</tr>
<tr>
<td>Coal – 16 (dry)</td>
<td>79.54</td>
<td>18.24</td>
<td>58.70</td>
<td>0.1947</td>
<td>0.0143</td>
<td>2.96</td>
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<tr>
<td></td>
<td>(±65.57)</td>
<td>(±8.09)</td>
<td>(±56.34)</td>
<td>(±0.1117)</td>
<td>(±0.0263)</td>
<td></td>
</tr>
</tbody>
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CONCLUSIONS

Examples in this paper proved that the suggested “Q-σ” Single Relaxation Model including the effect of one intrinsic process on the pressure dependence of acoustic quality factor can be applied well in practice. Although the results were satisfying for engineering problems (relative data distance smaller than 5%), further development seemed to be desirable. Thus, the new “Q-σ” Double Relaxation Model considers a second relaxation process connecting to a second internal phenomenon during the loading of the rock samples as well. Comparison of the results obtained from SRM and DRM confirmed that the new model provides a more precise description of the pressure dependence of acoustic quality factor. The relative data distances were almost two times smaller for this model.

ACKNOWLEDGMENTS

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## LIST OF SYMBOLS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>cov(m)</td>
<td>covariance matrix in parameter space</td>
<td>–</td>
</tr>
<tr>
<td>D</td>
<td>relative data distance</td>
<td>%</td>
</tr>
<tr>
<td>(d_i^{(c)})</td>
<td>calculated data at the i-th pressure value</td>
<td>–</td>
</tr>
<tr>
<td>(d_i^{(m)})</td>
<td>measured data at the i-th pressure value</td>
<td>–</td>
</tr>
<tr>
<td>(d\sigma)</td>
<td>stress increase</td>
<td>MPa</td>
</tr>
<tr>
<td>K</td>
<td>integration constant</td>
<td>–</td>
</tr>
<tr>
<td>N</td>
<td>number of measured points</td>
<td>–</td>
</tr>
<tr>
<td>M</td>
<td>number of model parameters</td>
<td>–</td>
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<tr>
<td>X</td>
<td>extensive quantity</td>
<td>–</td>
</tr>
<tr>
<td>(X_0)</td>
<td>extensive quantity at stress-free state</td>
<td>–</td>
</tr>
<tr>
<td>Q</td>
<td>quality factor</td>
<td>–</td>
</tr>
<tr>
<td>(Q_0)</td>
<td>quality factor at stress-free state</td>
<td>–</td>
</tr>
<tr>
<td>(Q_{\text{max}})</td>
<td>quality factor at maximum pressure</td>
<td>–</td>
</tr>
<tr>
<td>var</td>
<td>variance</td>
<td>–</td>
</tr>
<tr>
<td>(\Delta Q)</td>
<td>quality factor -drop at pressure (\sigma)</td>
<td>–</td>
</tr>
<tr>
<td>(\kappa)</td>
<td>proportionality factor</td>
<td>–</td>
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<tr>
<td>(\lambda)</td>
<td>relaxation parameter</td>
<td>1/MPa</td>
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<tr>
<td>(\sigma)</td>
<td>stress</td>
<td>MPa</td>
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## REFERENCES


